SOLVING NUMERICAL EQUATIONS OF HYDRAULIC PROBLEMS USING GENETIC ALGORITHMS AND EVOLUTION STRATEGY

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Abstract— This paper describes how to solve numerical equations of hydraulic problems that involve the calculation of free and forced channels. The problem is modeled by using the equation of Manning. The resulting numerical equation allows the calculation of outflows, inclination of the channels, roughness, loss of energy and other parameters that are function of the geometry of the channels. Since this equation presents high non-linearities and therefore it doesn't have a closed solution, we propose an alternative solution using Genetic Algorithms (*GAs*) and Evolution Strategy (*ES*).

Keywords— Genetic Algorithms; evolution strategy, channel design.

1 Problem description

According to the hydraulics theory, the behavior of the motion of fluids in forced and free conduits (channels) can be modeled based on the equations of Manning, Hazen-Willians, Darcy-Weisbach (Colebrook-Write), and Kutter (Chezy) (Chadwick and Morfett, 1994). These equations allow the calculations of outflows, inclinations of the channels, roughness, loss of energy and other parameters that are function of the geometry of the channels, e.g., circular, rectangular, triangular or trapezoidal. In this work, we use the Manning method whose general equation is described by:

$$d = \frac{c \cdot A^{1.67} \cdot s^{0.5}}{n \cdot P^{0.67}} \quad (1)$$

where d stands for discharge, c stands for a constant, A designates the channel area, s designates the channel slope, n is the Manning coefficient and P is the wet perimeter. For a channel with triangular section, we can write the following equations:

$$A = \left(\frac{1}{s_{rs}} + \frac{1}{s_{ls}} \right) \cdot \frac{h^2}{2} \quad (2)$$

$$P = h \cdot \left(\frac{\left(1 + s_{rs}^2\right)^{0.5}}{s_{rs}} + \frac{\left(1 + s_{ls}^2\right)^{0.5}}{s_{ls}} \right) \quad (3)$$

where s_{rs} is the right side slope, s_{lr} is the left side slope and the h is the channel depth. Using the International System, c is equal 1. Introducing equations (2) and (3) into the equation (1) and rearranging it results:

$$\frac{c.h^{2.67} s^{0.5}}{n.d} - \frac{\left(\frac{\left(1 + s_{rs}^2\right)^{0.5}}{s_{rs}} + \frac{\left(1 + s_{ls}^2\right)^{0.5}}{s_{ls}} \right)^{0.67}}{0.5.\left(\frac{1}{s_{rs}} + \frac{1}{s_{ls}} \right)^{1.67}} = 0 \quad (4)$$

The equation (4) presents high non-linearities and therefore it doesn't have a closed solution. So numerical methods may be employed to solve such a problem. In some cases, numerical methods present difficulties to find the optimal solution. In this paper, we propose an alternative solution using Genetic Algorithms (*GAs*) (Michalewicz, 1996) and Evolution Strategies (*ES*) (Beyer, 1995). *GA* and *ES* are search and optimization algorithms based on the principle of natural evolution and population genetics, which have been applied successful to many engineering problems in different knowledge areas. The basic idea is to maintain a population $P(t):\{x_1(t), \ldots, x_i(t), \ldots, x_\mu(t)\}$ of individuals (candidate solutions) which evolve under selective pressure that favors better solutions. The interest in Evolutionary Algorithms is increasing very fast, because their robust and powerful adaptive search mechanisms. Evolutionary algorithms have been used to dealing with multi-dimensional and multi-modal search problems. In the following, we describe the two evolutionary algorithms.

2 Genetic Algorithms

GAs maintain a population of individuals which are generated at random. Each individual $x_i(t)$ represents a potential solution of the problem at hand. Individuals are evaluated to give a measure of their fitness. Then, a new population, iteration t+1, is formed by selection of the more fit individuals (selection step). Some individuals of the new population undergo transformations due to crossover and mutation operators to form new solutions (Michalewicz, 1996). After a certain number of generations the search converges and, if successful, the fittest individual represents the optimal solution. *GAs* in its simplest form uses three operators: selection, crossover and mutation.

Selection: In the selection step, individuals are chosen to participate in the reproduction of new individuals. In this paper, the simple and robust tournament selection is used. This method consists of a random selection of two or more individuals. The individual with the highest fitness advances to the next generation.

Crossover: The crossover operator combines characteristics of two parent individuals to form two offspring. In this paper arithmetical crossover is used. Let $x_1(t)$ and $x_2(t)$ be two individuals to be crossed. Then the two offspring $x_3(t+1)$ and $x_4(t+1)$ are produced as a linear combination of their parents $x_1(t)$ and $x_2(t)$, i.e.,

$$x_3\ (t+1) = a\, x_1(t) + (1-a)\, x_2(t) \qquad (5)$$

$$x_4\ (t+1) = (1-a)\, x_1(t) + a\, x_2(t) \qquad (6)$$

where $a \in [0,1]$. The crossover operator is applied with a probability p_c.

Mutation: The mutation operator alters one or more genes of a individual by a random change. Let the individual $x_i(t) = x_1(t), \ldots, x_j(t), \ldots, x_m(t)$, and the gene x_j to be selected for mutation. The domain of the variable x_j is given by $x_j = \lfloor \underline{x}_j\ \overline{x}_j \rfloor$, where $\underline{x}_j$ and $\overline{x}_j$ denote the lower and upper bound of the variable x_j, respectively. Then the result of the application of this operator is $x_i(t) = x_1(t), \ldots, \tilde{x}_j(t), \ldots, x_m(t)$, where $\tilde{x}_j$ is a random value (uniform probability distribution) within the domain of x_j. The mutation operator is applied with a probability p_m.

3 Evolution Strategy

Evolution Strategy represents each individual as a real-valued vector. Its main operator is mutation, and it allows self-adaptation of strategy parameters through standard deviation and covariance (Beyer, 1995). Comparable to other optimization techniques, the performance of *ES* depends on a suitable choice of internal strategy control parameters. Apart from a fixed setting, *ES* facilitate an adjustment of such parameters within a self-adaptation procedure while in *GAs* the control parameters are adjust by trial and error method.

The self-adaptation of strategy parameters provides one of the main features of the success of *ES*, because *ES* use evolutionary principles to search in the space of object variables and strategy parameters simultaneously. In the (μ+λ)-*ES* case, the parental generation is taken into account during selection, while in the (μ,λ)-*ES* case only offspring undergoes selection, and the parent die off. In this paper, a derandomized (1+λ)-*ES* with individual step-sizes and correlated mutations is employed (Ostermeier *et al.*, 1993).

The implemented (1+λ)-*ES* try to derandomize the adaptation process by exploiting information gathered in preceding iterations. Derandomized adaptation usually takes place without mutation of the strategy parameters itself. It uses selected points (selected mutation steps) in object parameter space for strategy parameter adjustment. The derandomized (1+λ)-*ES* has the start point chosen randomly with uniform probability distribution, and it use two operators: selection and mutation. The recombination operator is not utilized because μ=1 is adopted.

Selection: The selection (adaptation) operator is completely deterministic; it just chooses the best fitted μ individuals ($1 \le \mu < \lambda$) out of the set of λ offspring individuals. The individual (μ=1) with the highest fitness advances to the next generation, i.e.,

$$x_\mu(t+1) = x_{\lambda sel}(t) \qquad (7)$$

where $x_\mu(t)$ is the parameter vector of generation t, λsel marks the quantities of the *sel*ected offspring of generation t.

Mutation: The mutation operator is the main operator and introduces random modification into the population. The implemented algorithm realizes mutation ellipsoids in the direction of the selected offspring. The creation of λ offspring is given by:

$$\boldsymbol{x}_{\lambda i}(t+1) = \boldsymbol{x}_{\mu}(t) + \delta_{scal}(t)\cdot(\xi_i \delta_{iso}(t) Z_i + \psi_i \delta_{ani}(t) A_0(t)) \quad (8)$$

where i=[1;λ]; $\delta_{scal}(t)$ are scaling factors of generation t, ξ_i is the step-size changing factor (1,0.1)-normally distributed, $\delta_{iso}(t)$ is the step-size of *iso*tropic mutations of generation t; Z_i is the random vector with components (0,1)-normally distributed, ψ_i is (0,1)-normally distributed, $\delta_{ani}(t)$ is the step-size of anisotropic (correlated) mutations of generation t, $A_0(t)$ is the orientation of long axis of mutation ellipse in generation t (Ostermeier *et al.*, 1993). The domain of the variable x_j (object variable vector) is given by $x_j = \lfloor \underline{x}_j \; \bar{x}_j \rfloor$, where $\underline{x}_j$ and $\bar{x}_j$ denote the lower and upper bound of the variable x_j, respectively.

4 Results

To illustrate the use of these evolutionary algorithms, we show the calculation of the right side slope (s_{rs}) for a triangular channel according the Manning method, which is described by the equation (4). So we define the function f by:

$$f = \frac{c.h^{2.67} s^{0.5}}{n.d} - \frac{\left(\frac{\left(1+s_{rs}^2\right)^{0.5}}{s_{rs}} + \frac{\left(1+s_{ls}^2\right)^{0.5}}{s_{ls}} \right)^{0.67}}{0.5\left(\frac{1}{s_{rs}} + \frac{1}{s_{ls}} \right)^{1.67}} \quad (9)$$

and the fitness function is calculated according to the inverse of the absolute value of the function f. The parameters used for the simulation are given in the table1.

Table1. Simulation parameters.

Variable/ Parameter	Description	Value
d	discharge	0.5 [m^3/s]
c	constant	1
n	Manning coefficient	0.012
s	channel slope	0.005 [m/m]
h	depth	0.5 [m]
s_{ls}	left side slope	1.0 [m/m]
s_{rs}	right side slope	? [m/m]

For the *GA* implemented, we used the real representation. Each individual is codified as a vector of numbers in floating point. In this case, the individual consists of only one gene (the right side slope). The lower and upper bounds of s_{rs} specified by the designer was the interval $[0, 5]$. The *GA* was initialized randomly with a population size $\mu = 50$, crossover probability $p_c = 0.6$, crossover parameter a = 0.5, and mutation probability p_m = 0.05.

For the derandomized (1+λ)-*ES* implemented, a population size of $\mu = 1$ parent and $\lambda = 50$ offspring was used. The selection and mutation operators applied are as described in previous section.

The optimal value for the right side slope obtained by using *GA* and *ES* is s_{rs} = 0.89558. Figure 1 deploys the convergence of f for both *GA* and *ES*.

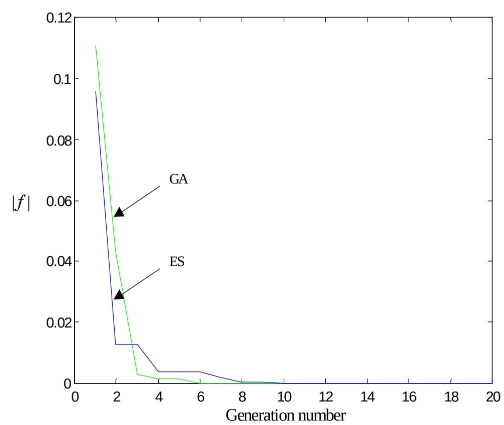

Figure1. Convergence of the *GA* and *ES*.

5 Conclusions

This paper presents an alternative approach to solve numerical equations of hydraulic problems based on genetic algorithms and evolution strategy. The problem was described by using the Manning equation. For the solution of the numerical problem appropriate genetic operators and a fitness function were employed. The validity of the solution was demonstrated by a simulation example. As we can note from the figure 1, *GA* as well as *ES* found out the value for the right side slope in less than 10 generations. The obtained results were equals, showing the feasibility of both evolutionary techniques. As the same way, you may determine the other design variables, e.g., discharge, etc. The results so far demonstrated clearly the potential of these techniques to solving this kind of problem. In future works, we plan to extend this one to cover more design variables simultaneously giving so the possibility of interaction with the designer, since more than one feasible solution can be found.

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