X-OVER PARAMETER CONTROL FOR A GA-OPTIMIZER DEDICATED TO EIGENSTRUCTURE ASSIGNMENT/LQR DESIGNS - PART II - COMPUTATIONAL SIMULATIONS AND PERFORMANCE ANALYSIS

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Abstract— The main objective of this article is to present computational simulations results and performance analysis of the proposed crossover operations parameters adjustment methods for a parallel multiobjective genetic algorithm (*PMOGA*) dedicated to dynamic systems eigenstructure assignment. The proposed model development for crossover adjustment is presented in a first paper. The computational simulation results are performed on high performance parallel computers and the analysis considers three result types: proposed methods comparison, solutions evolution and state feedback controllers quality.

Key Words— Genetic algorithm, eigenstructure assignment, parameters tuning , LQR designs, decision-making unit, crossover operator, parallel computation.

1 Introduction

The most fundamental effort in this work is the determination of state feedback controllers via *LQR* design, ie, the final product that results from this effort are controllers that have the ability to perform eigenvalues and eigenvectors assignment. Several steps, such as: the control problem formulation and *GA*-optimizer aspects, have to be taking into account before the controllers determination otherwise it is not possible to find out feedback laws that satisfy the required eigenstructure. The *GA* optimizer, specifically, the crossover (*X-OVER*) operation parameters, that can work as a mechanism for the search guidance, is a relevant matter to reach a satisfactory final result.

The performance of the proposed method was verified on state space variable linearized model that represents the dynamics of an aircraft. The computational simulation results were performed on parallel computational environment.

The proposed parameter adjustment methods and the best solutions' evolution, that is a function of *LQR* designs' Q and R weighting matrices, whose search is performed by *GA* optimizer, are analysed together, ie, comparing the parameter adjustment influences on the search performance. The second analysis considers the performance of the controllers when they are implemented on state variable model and are subjected to an impulse signal perturbation.

The following four sections discuss the computational simulations and performance analysis of the proposed methods. The first one, makes a comparative analysis between the methods' action and the *PMOGA* search. The second one, presents the proposed method efficiency by exhaustive parallel simulations. The third one, verifies the state feedback controllers performance. The last one, gives the concluding remarks of this work development in a global vision.

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2 *X-OVER* parameter actions

This section presents a comparative analysis of the *X-OVER* operations parameter adjustment influences on the performance of the *PMOGA* search. The *PMOGA* could not present feasible solutions, for the selected cases, if the proposed methods actions were not utilized to improve the Q and R matrices search.

The master task final population profiles for the quasi-dynamic method and for the dynamic methods (deterministic and adaptive rules) are presented in Fig.1. The quasi-dynamic method did not help to find a feasible controller, Fig.1a. The adaptive rule, Fig.1b for $\Delta q(t) \neq 0$ and $\Delta r(t) \neq 0$ helped the *GA*-optimizer to find weighting matrices that lead to a whole family of feasible controllers, whose worst sensitivity, Δ_j^{max}, $j = 1, \ldots, N$, lies on between 0.79 and 0.98; but for this rule with two variations, $\Delta q(t) = 0$ or $\Delta r(t) = 0$, no feasible controllers were found for a search cycle of 100 generations.

The dynamic right deterministic rules, Fig. 1c, improved the Q and R matrices search, for two variation set: ($\Delta q(t) \neq 0$ and $\Delta r(t) \neq 0$) and ($\Delta q(t) \neq 0$ and $\Delta r(t) = 0$). For this rule six feasible controllers were determined for each set of variations, whose controllers Δ_j^{max} lie on between $0.90 - 0.97$ and $0.91 - 0.97$, respectively. No feasi-

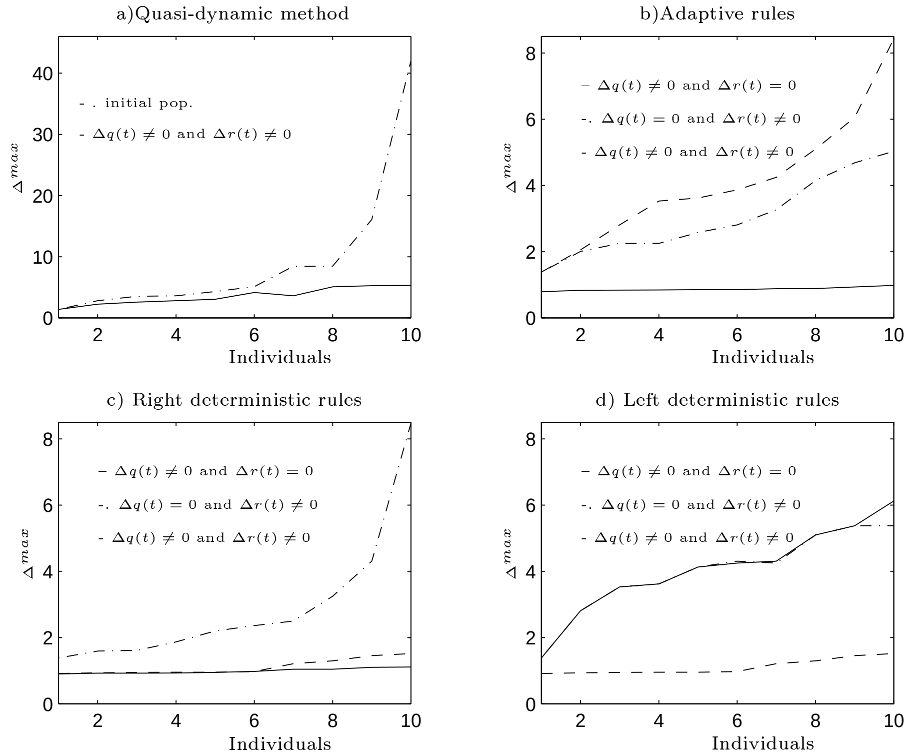

Figure 1. Parameter setting actions on the master task final population profile

ble controllers were obtained for the following set: ($\Delta q(t) = 0$ and $\Delta r(t) \neq 0$).

The left deterministic rule, Fig.1d, presented six feasible controllers for its variation, $\Delta q(t) \neq 0$ and $\Delta r(t) = 0$, but for the other two no feasible controllers could be determined. All rules types and its variations improved the final populations profiles.

Fig.'s 2 to 11 present the effects of the proposed parameter setting control rules for a hard

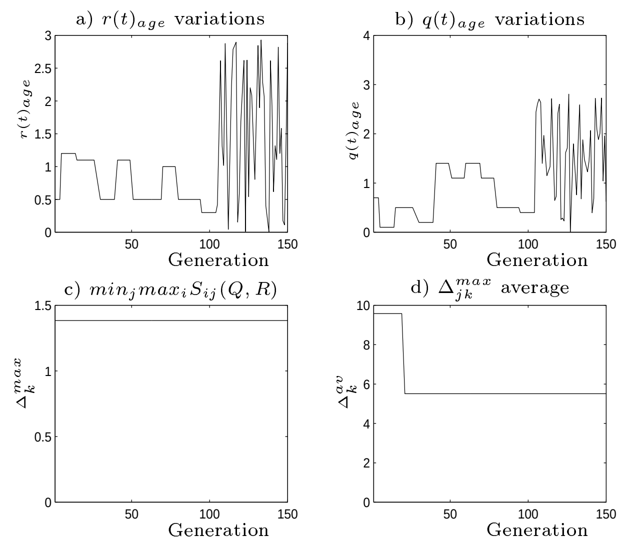

Figure 2. Quasi-dynamic Method - *X-over* operation parameters variation, maximum sensitivity and average sensitivity *versus* search cycle k iterations.

convergence case that the quasi-dynamic method, Fig.2, did not improve the search for ten CPU's parallel processing, ie, the $PMOGA$ could not find out Q and R matrices that lead to feasible controllers. Each figure presents the master task results and it consists of four curves; types a and b curves show the $q(t)_{age}$ and $r(t)_{age}$ parameter changes, respectively; types c and d curves show the best individual maximum sensitivity for each k generation step, Δ_k^{max}, that is given by $min_j max_i s_{ij}(Q, R)$ ($i = 1, \ldots, n$ and $j = 1, \ldots, N$), where n is the dynamic system order and N is the number of individuals in the GA-optimizer population, and the worst maximum sensitivity average, Δ_k^{av}, of the permanent population that is given by $(\sum_j^N max_i s_{ij}(Q, R))/N$, respectively.

The adaptive rule and its main element that directly influenced the off-springs formation are the variation steps, $\Delta q(t)$ and $\Delta r(t)$, and this rule improved the search for $\Delta q(t) \neq 0$ and $\Delta r(t) \neq 0$, Fig. 3. Fig. 3a and Fig. 3b curves are related with some QR-individuals displacements to new quadrants in the search space and at the beginning of each new change point the GA- optimizer has a chance to perform a local exploration of the quadrant or even stay in this region until the newest change point appears, (Bottura and Neto, 2001). After a certain number of generations, by the end of the search cycle, the $q(t)_{age}$ and $r(t)_{age}$ parameters only accept small random variations, ie, this policy allows to concentrate the search on a limited region of the solution space. Fig.3c shows that around half of the search cycle, the first feasible controller was obtained and for the rest of the search cycle all individuals of the permanent population are assembled by feasible controllers, Fig. 1b. The Δ_{jk}^{max} average, Fig. 3d, was improved before the first quarter of the search cycle, as for quasi-dynamic method, Fig.2d, and the most significant improvement was obtained after the 100^{th} generation. For $\Delta q(t) = 0$ or $\Delta r(t) = 0$, Fig.'s 4 and 5, no feasible controllers were presented.

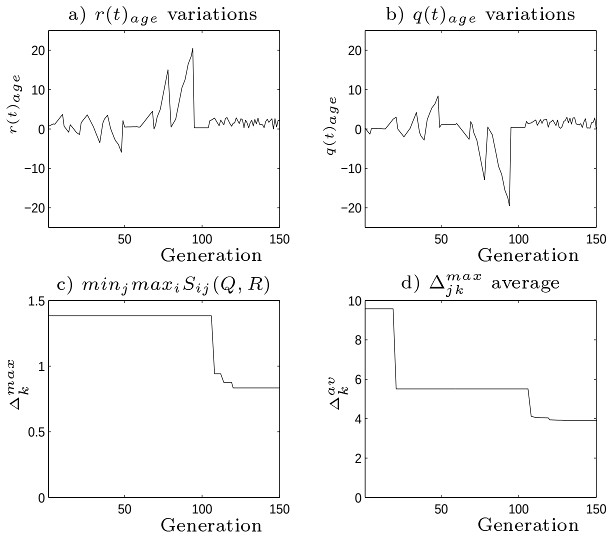

Figure 3. Adaptive rule for $\Delta q(t) \neq 0$ and $\Delta r(t) \neq 0$ - *X-OVER* operation parameters variation, maximum sensitivity and average sensitivity *versus* search cycle k generations

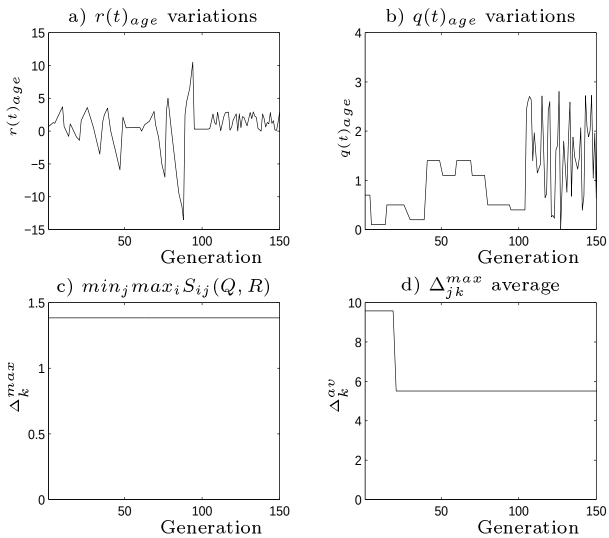

Figure 4. Adaptive rule for $\Delta q(t) = 0$ and $\Delta r(t) \neq 0$ - *X-OVER* operation parameters variation, maximum sensitivity and average maximum sensitivity *versus* search cycle generations

Fig.'s 6 to 8 present the behavior of the dynamic right deterministic rules and its effects on the weighting matrices search. An analysis, similar to the one conducted for the adaptive rule can be carried out for the other rules. As can

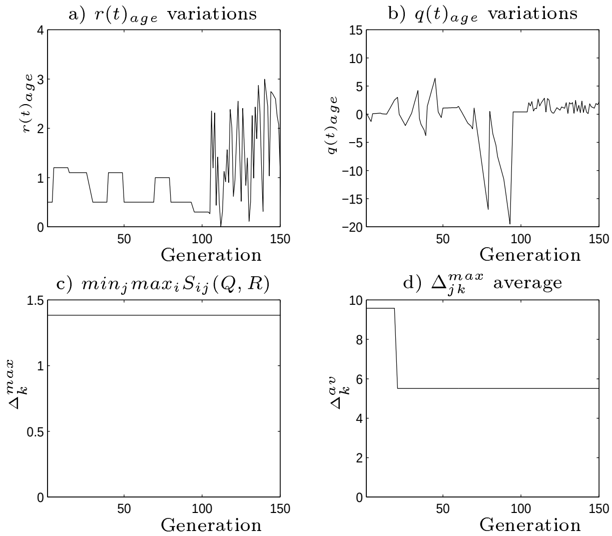

Figure 5. Adpative rule for $\Delta q(t) \neq 0$ and $\Delta r(t) = 0$ - *X-OVER* operation parameters variations, maximum sensitivity and average maximum sensitivity *versus* search cycle generations

be seen this rule presented feasible controllers for $\Delta q(t) \neq 0$ and $\Delta r(t) \neq 0$, Fig. 6c, and $\Delta q(t) \neq 0$ and $\Delta r(t) = 0$, Fig. 6c, respectively; but the second case presented a feasible controller for fewer generations of the search cycle than the adaptive rule. For the case $\Delta q(t) = 0$ and $\Delta r(t) \neq 0$, no feasible controllers were presented or even any kind of improvement on the best individual of the initial population, besides the improvements obtained for the maximum sensitivity average of the entire population.

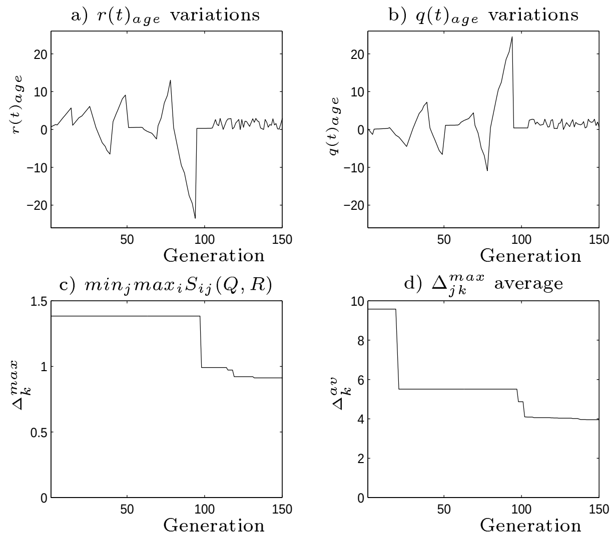

Figure 6. Right direction deterministic rule for $\Delta q(t) \neq 0$ and $\Delta r(t) \neq 0$ - *X-OVER* operation parameters variation, maximum sensitivity and average sensitivity *versus* search cycle generations.

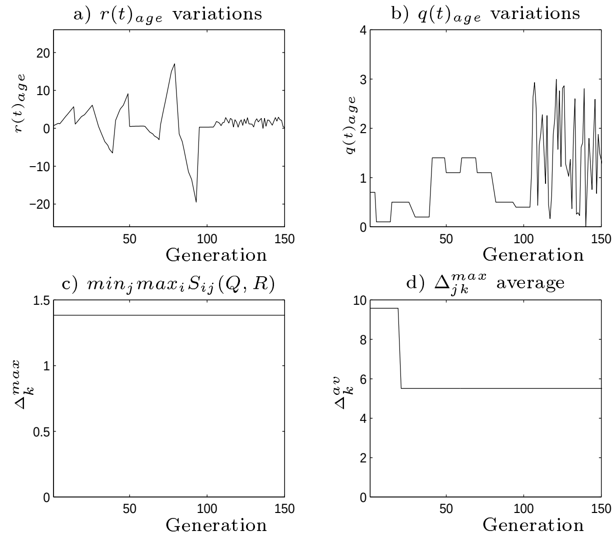

Figure 7. Right direction deterministic rule for $\Delta q(t) = 0$ and $\Delta r(t) \neq 0$ - *X-OVER* operation parameters variation, maximum sensitivity and average sensitivity *versus* search cycle generations

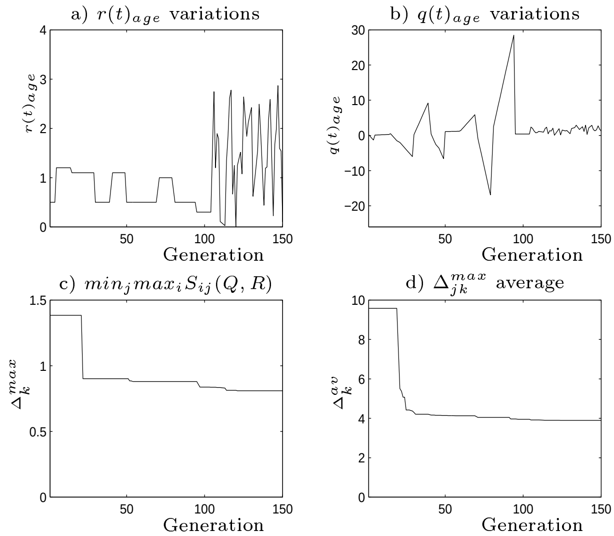

Figure 8. Right direction deterministic rule for $\Delta q(t) \neq 0$ and $\Delta r(t) = 0$ - *X-OVER* operation parameters variations, maximum sensitivity and average sensitivity *versus* search cycle generations.

The dynamic left deterministic rule results are presented in Fig.'s 9 to 11. Only the case, for $\Delta q(t) \neq 0$ and $\Delta r(t) = 0$, Fig. 11, presented improvements on the search and the final permanent population is assembled by six feasible controllers, Fig.11d; the first feasible controller was obtained around the 50^{th} generation, Fig.11c. The other two cases, Fig.'s 9 and 11, did not present any improvements on the search cycle generations, besides the improvements provided to final population profile.

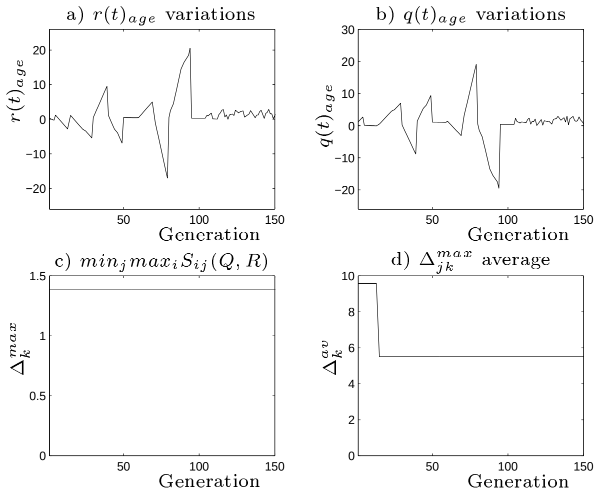

Figure 9. Left direction deterministic rule for $\Delta q(t) \neq 0$ and $\Delta r(t) \neq 0$ - *X-OVER* operation parameters variation, maximum sensitivity and average sensitivity *versus* search cycle generations.

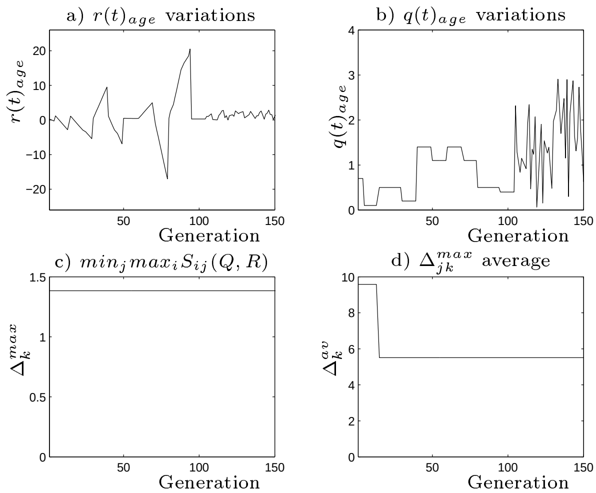

Figure 10. Left direction deterministic rule for $\Delta q(t) = 0$ and $\Delta r(t) \neq 0$ - *X-OVER* operation parameters variation, maximum sensitivity and average sensitivity *versus* search cycle generations.

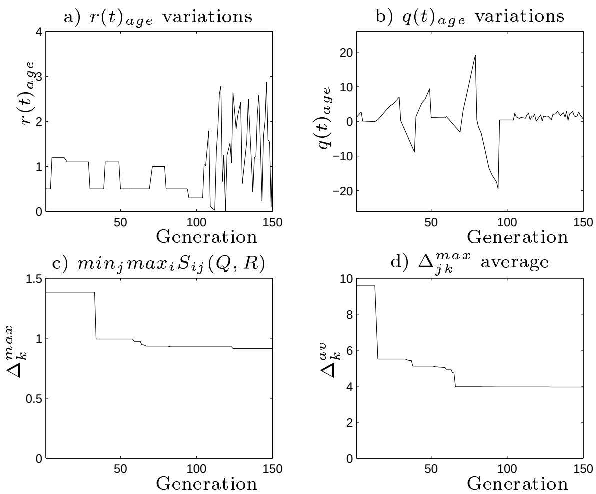

Figure 11. Left direction deterministic rule for $\Delta q(t) \neq 0$ and $\Delta r(t) = 0$ - *X-OVER* operation parameters variation, maximum sensitivity and average sensitivity *versus* search cycle generations.

Case	Search quantity		Success
	Minimum	Maximum	quantity
1	2	110	4
2	76	94	6
3	3	143	7
4	29	90	6
5	78	118	6
6	25	25	1
7	60	183	4
8	28	150	6
9	4	64	5
10	53	125	8

Table 1. Adaptive parameter setting performance results.

Case	Search quantity		Success
	Minimum	Maximum	quantity
1	59	94	3
2	64	101	8
3	35	136	8
4	35	125	8
5	83	180	8
6	62	150	3
7	52	189	6
8	55	186	5
9	35	105	7
10	37	186	8

Table 2. Dynamic left deterministic parameter setting performance results.

Case	Search quantity		Success
	Minimum	Maximum	quantity
1	3	134	7
2	73	125	8
3	3	163	8
4	19	195	7
5	28	93	5
6	19	42	3
7	72	185	6
8	28	189	7
9	4	107	8
10	28	115	8

Table 3. Dynamic right deterministic parameter setting performance results.

3 Exhaustive Simulations

The exhaustive simulation results are presented on Tables 1 to 3. These tables show computational simulation results from ten cases for the same dynamic system, each case corresponds to different initial conditions and the $PMOGA$ could not produce any feasible solution when the parameters did not suffer any adjustment during the search cycle for the quasi-dynamic method, but solutions were found when the proposed dynamic methods were activated considering $\Delta q(t) \neq 0$ and $\Delta r(t) \neq 0$. For each case, eight processors were used, this means that 240 simulations were performed to validate the proposed methods on helping the GA-optmizer in the Q and R matrices search that satisfies the desired EA. These tables present synthesized results, the minimum and the maximum generations number of the search to find out a feasible solution, as well as the number of successful searches among 8 trials.

Comparing the performance results of the three rules, Tables 1 to 3, it can be verified that the adaptive rule led to the first feasible solution and the majority of its feasible results happened before the 100^{th} generation of the search cycle, this means that this rule is more effective than the other two. The dynamic deterministic method produced more successful results than the adaptive rule. Considering the deterministic method, its right rule has presented a greater number of successful results and were produced for a lesser number of generations.

4 Controllers Performance

The controllers obtained from the exhaustive simulations, section 3, have been implemented on the linearized state variable model and an impulse input has been applied to verify their performance. This procedure test was performed for each hard case presented on Tables 1 to 3 and for families of controllers obtained from just one processor. The impulse simulation results were compared with a basic controller, (Davis and Clarke, 1995), and it was observed that the controllers presented good performance for this kind of perturbation.

Three controllers performance, obtained with the help of the adaptive method guidance, are compared with the basic controller (*Bcontroller*) performance, Fig. 12. As can be observed, task 07 controller performance, Fig. 12d, is the one that presented best response when compared with the basic, master and task 06 controllers. The master and task 06 controllers presented a small overshoot compared with *Bcontroller*, but they present a greater settling time than the basic controller.

The impulse signal response for a controller family is presented in Fig. 13. In this case all controllers were obtained from the same processor task and the closed-loop dynamic system impulse responses are very similar for controllers whose worst sensitivity is not very close, as can be seen in Fig.'s reffig-s408a) and b), for these controllers the worst eigenvalues sensitivities are 0.73 and 0.84, respectively. The system impulse response for controllers 09 and 14 implementation, Fig's 13c) and d), are more damped than controllers 01 and 04 responses and for these controllers the worst eigenvalues sensitivities are 0.90 and 0.99, respectively. Considering that these controllers originated from the same task, we can conclude that as the controller worst sensitivity is near

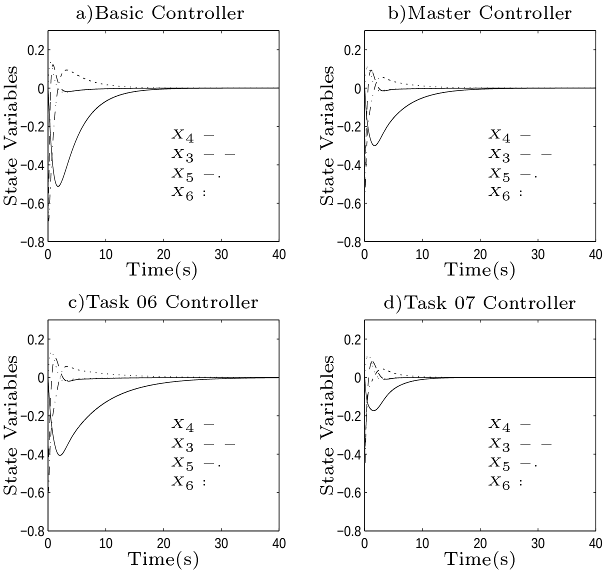

Figure 12. Case I - Impulse Response - Controllers obtained with Adaptive rule

1 (one) the transient responses present a closer damped response.

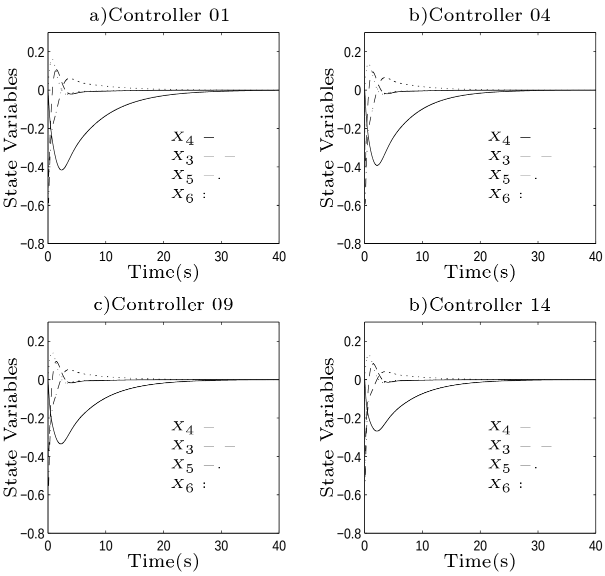

Figure 13. Case I - Impulse Response for a Task 09 - Controller family Controllers obtained with Adaptive rule.

5 Concluding Remarks

This work has presented the performance of methods for parameter adjustment of the crossover operation, the main purpose of this parameter is to control the degree of combination between two individuals. The methods were computationally implemented as rules on a parallel multiobjective genetic algorithm, specifically designed to search Q and R weighting matrices of the Linear Quadratic Regulator Problem, LQR. A control law, obtained from LQR design, assigns the specified eigenstructure in a dynamic system and it efficiency has been verified for cases that presented no feasible solution, when the proposed methods were not implemented.

In general, all the three rules have presented improvements on the *PMOGA* search, because they have the ability to guide the search that leads to a feasible eigenstructure assignment. The adaptive rule has shown to be the most efficient, because most of its results were obtained with lesser generations of the search cycle.

The parallel design can become more efficient, to increase the number of feasible solutions, if each task assumes different rules for the *X-OVER* operation parameters variation.

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