USING LINEAR ARX TERMS IN RBF NETWORK MODELS: A CASE STUDY EMPLOYING A THERMAL PROCESS

Gladstone B. Alves and Marcelo V. Corrêa

Curso de Engenharia Elétrica
Centro Universitário do Leste de Minas Gerais — Unileste-MG
Av. Tancredo Neves 3500, Cel. Fabriciano, MG, Brazil
E-mails: gladston@unilestemg.br, mcorrea@unilestemg.br

Luis A. Aguirre

Programa de Pós Graduação em Engenharia Elétrica
Universidade Federal de Minas Gerais
Av. Antônio Carlos 6627, 31270-901 Belo Horizonte, M.G., Brazil
E-mail: aguirre@cpdee.ufmg.br

Abstract— This paper considers two different types of radial basis function (RBF) model structures, namely the traditional topology and an augmented topology that includes linear polynomial terms of the ARX (*autoregressive with exogenous inputs*) type. In particular, the aim is to investigate the potential advantages in the use of linear terms in nonlinear system identification problems using real data collected from a simple thermal process. The inclusion of linear terms in RBF network models in the example discussed had two main advantages: (1) the resulting model structures were more parsimonious, and (2) such models, in general, presented better steady-state performance.

Key Words— Nonlinear system identification, RBF networks, Model structure selection

1 Introduction

One of the most challenging issues in nonlinear system identification is the choice of an adequate model structure for a given problem. Although this question has recently attracted some attention (Kadtke et al., 1993; Mao and Billings, 1997; Aguirre et al., 2000; Henrique et al., 2000) it seems far from having a definitive answer.

It is known that if the model structure is overparameterized it may show spurious dynamical regimes and even become unstable (Aguirre and Billings, 1995). One of the possible reasons for the difficulty to adequately select the structure of a nonlinear model is that decisions are usually made based on criteria that do not cater for the overall performance of the model. In this sense, such models could present poor generalization on data not used in the estimation phase. Some recent contributions to the structure selection problem has been made using concepts of term cluster and cluster coefficients for nonlinear polynomial and rational models (Mendes, 1995; Aguirre, 1997). Such concepts may provide useful information concerning fixed points, symmetry and nonlinear static relationships within the systems. In this sense, some results suggest that is possible to improve model global stability through the use of *a priori* information about the system under investigation in the structure selection of nonlinear models (Aguirre et al., 2000).

Artificial neural networks (ANNs) have been used in recent years in some system identification problems. However, the application of ANNs to modeling and analyzing nonlinear systems present even stronger drawbacks concerning structure selection. In order to reach better mapping accuracy it is a common procedure to increase network complexity allocating multiple hidden layers or simply increasing the number of nodes in a single hidden layer. This approach usually results in network over-fitting, poor generalization performance and slowness of parameter estimation, especially when using nonlinear optimization techniques. A methodology for network structure determination particularly helpful in system identification performs the pruning of large trained ANNs (Reed, 1993). More recently proposed pruning algorithms demonstrate the advantages of obtain parsimonious network models (Henrique et al., 2000).

A promising network model representation is the radial basis function networks (RBFNs). RBFNs are very attractive because of the simplicity of the structure that is linear in the parameters, unlike other types of ANNs. Therefore, RBFN models are compact and very easy to deal with, since standard linear estimation techniques can be applied. Moreover, some locality properties make RBFNs preferable to multilayered networks when issues such as online adaptability are concerned. A common practice in some applications of RBFNs is to add a linear polynomial to the conventional network topology, in order to improve approximation performance and to enhance the rate of convergence in network learning (Poggio and Girosi, 1990). Other researchers suggest other reasons for the use of the linear polynomial. (Knohl and Unbehauen, 1998) assure that the use of linear polynomial terms in RBFN models could avoid unwanted oscillations between the nodes in adaptive control applications. (Mees, 1993) suggests that augmented basis functions behave much better in the presence of the linear polynomial terms. However, apparently there is no comprehensive study about the independent effect of the linear terms in system identification applications. In this respect, this paper focuses on the comparative performance of RBFN models augmented with linear polynomial terms, in the light of real data generated from a thermal process.

This paper is organized as follows. Section 2 introduces RBFNs and the two types of model structures considered in this work: traditional network and topology with added linear polynomial terms. Section 3 briefly describes the thermal process that generated the data used in this comparative study. In Section 4 the performances of models obtained from real data are compared. Models with the different structures are also compared concerning the ability to approximate the system nonlinear static relationship between input and output. The correct fit to the static curve is very important for control purposes, especially for applications such as the control of chemical processes, since information about changes in steady-state gain can be obtained from this curve (Henrique et al., 2000). Moreover, it is believed that the quality of fit to the static function is an important tool of validation since it reflects in the generalization of the model. Finally, Section 5 summarizes the main points of the paper.

2 Radial Basis Function Networks

The Radial Basis Function (RBF) approximation method was traditionally used for strict interpolation in a multidimensional space. There are many possible RBFs, generally divided into functions with global or local properties. Local RBFs interpolate only in a region of input space around its center, whereas global RBFs extrapolate globally. Examples of local and global RBFs are, respectively:

- Gaussian: $\phi(r) = e^{-r^2/\sigma^2}$,
- Thin-plate spline: $\phi(r) = r^2 \log(r)$,

where σ is the receptive width of the locally-tuned Gaussian basis function which describes the sharpness of the hyperbolic cone used in the RBF, and r is the function argument.

Some results have demonstrated that the choice of the basis function is not particularly crucial for the performance of the approximation scheme although reasonable advantages can be derived in the use of a specific RBF (Jackson, 1988a). Relative performance of different types of RBFs have been surveyed in (Jackson, 1988b; Carlin, 1992).

Given a set of samples $(\mathbf{x}_j, y_j, j = 1, \dots, N)$, a single-output RBF network, implementing a mapping $f : \Re^n \longrightarrow \Re$, can be represented as (Zhu and Billings, 1996):

$$f(\mathbf{x}) = w_o + \sum_{i=1}^{m} w_i \phi(\|\mathbf{x} - \mathbf{c}_i\|), \qquad (1)$$

where $w_o \in \Re$ is a constant offset, $\mathbf{x} \in \Re^n$ is the input vector, $\phi(\cdot)$ is an RBF from $\Re^+ \longrightarrow \Re$, $\|\cdot\|$ denotes an Euclidean norm, $\mathbf{c}_i \in \Re^n$ $(i = 1, \dots, m)$ are the RBF centers and $w_i \in \Re$ $(i = 1, \dots, m)$ are the output weights. In order to capture system dynamics the RBF input vector $\mathbf{x}$ must be represented as a set of lagged input and output signals

$$\begin{aligned} \mathbf{x}(k) = [&y(k-1) \ \dots \ y(k-n_y) \\ &u(k-1) \ \dots \ u(k-n_u)]^{\mathrm{T}}, \end{aligned} \qquad (2)$$

where $\mathbf{x}(k)$ is a vector with dimension $n = n_y + n_u$, and n_u and n_y are lags of input and output signals, respectively. In this context, an excitation $\mathbf{x}(k)$ produces a network output $y(k)$.

In system identification it is a common procedure to add linear AR (Autoregressive) terms, as well as input terms, on the right-hand side of Equation (1) (Sze, 1995), which leads to an RBF model structure of the form (for the SISO case)

$$\begin{aligned} y(k) = w_0 + &\sum_{j=1}^{n_{u\ell}} w_j u(k-j) + \\ &\sum_{j=1}^{n_{y\ell}} w_{n_{u\ell}+j} y(k-j) + \\ &\sum_{j=1}^{m} w_{n_{u\ell}+n_{y\ell}+j} \phi(\|\mathbf{x}(k) - \mathbf{c}_j\|) + \\ &e(k), \end{aligned} \qquad (3)$$

where $n_{u\ell}$ and $n_{y\ell}$ are lags of linear input and AR terms respectively, $\mathbf{x}(k)$ is the RBF input vector, defined as Equation (2), and $e(k)$ is the noise.

The aim of this work is to compare the different RBF model structures represented by Equations (1) and (3), which means to give some insight about the independent effect of the linear polynomial terms in nonlinear system identification problems. Figures 1(a) and 1(b) shows diagrams of these types of RBF model structures. In particular, special emphasis will be given to the problem of recovering the nonlinear static function of the system.

Consolidated methods used to perform structure selection and parameter estimation for other

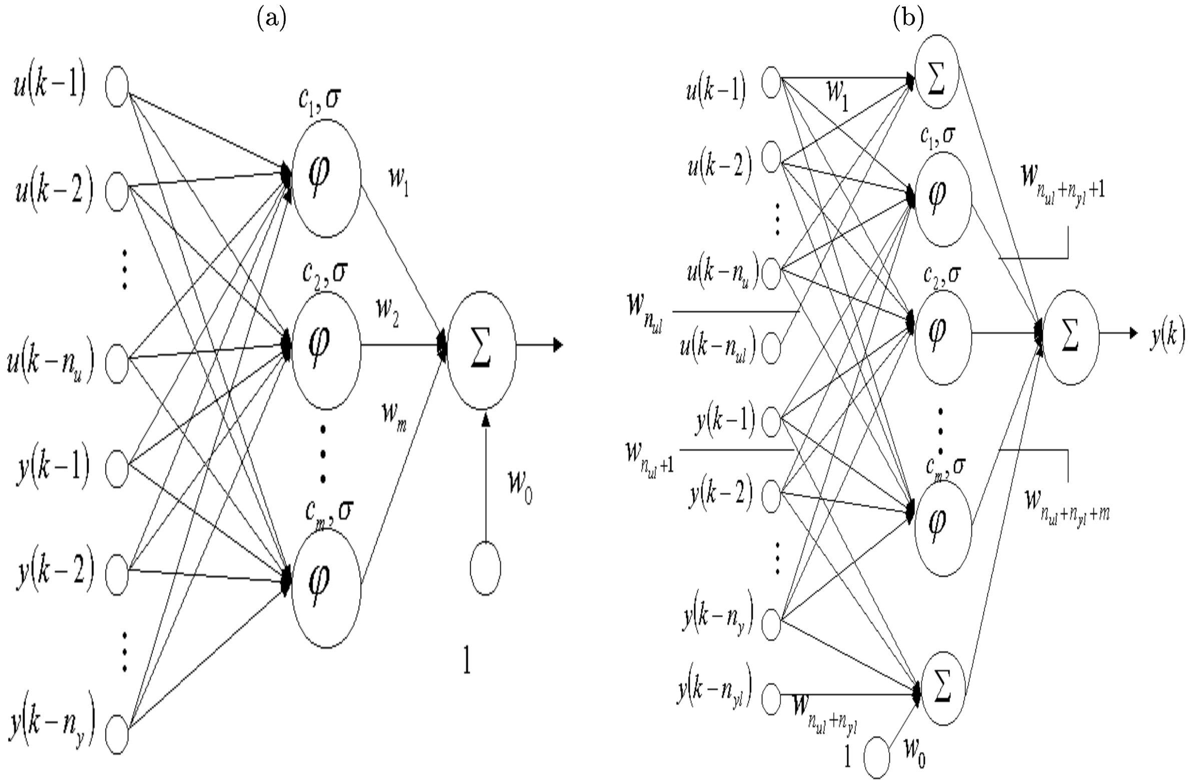

Figure 1. RBF model structures compared in this study. (a) Traditional structure, from Equation (1). (b) RBF structure with added ARX terms, from Equation (3). In this case, it was considered that $n_{ul} > n_u$ and $n_{yl} > n_y$.

linear-in-the-parameter representations (Aguirre, 2000) can be readily extended to the RBF model representation. This is one of the main justifications for the use of the RBF model representation in nonlinear system identification problems. A commonly used procedure is selecting the RBF centers (structure selection) from the input data and determining the output weights (parameter estimation) simultaneously, by means of the Error Reduction Ratio (ERR) criterion (Zhu and Billings, 1996). To avoid an excessive number of RBF terms in the model (Equation (3)), a subset selection procedure can be performed, based on the Orthogonal Least Squares (OLS) method (Chen et al., 1991; Mees, 1993). However, throughout this paper the centers are determined by a k-means clustering algorithm (Spath, 1980), and the weights estimated by standard least squares. The RBFs chosen for the hidden layer nodes, $\phi(\cdot)$, are Gaussian functions, with an appropriate width value determined by trial and error procedure.

3 Thermal Process and Data Description

This study considers real data measured from a simple thermal process. This process consists of a small electrical furnace without thermal isolation. The inner temperature is measured by an NTC resistor connected with an Wheatstone bridge, which is balanced at room temperature. The process is sensitive to external disturbances, such as fluctuations in the outdoor temperature and air currents (Abreu, 1993). Four independent data sets were collected in different days, with different input (electrical power) excitations, namely quantified noise and step functions. Figure 2 shows a plot of the data set used in the identification - $frq2$ - which contains 83 samples. The other three data sets - $frq1$, $fd1$ and $fd2$ - were used in the dynamic validation of the models. The sampling time used was $T_s = 210s$. For further descriptions of the experiment design see (Rodrigues, 1996).

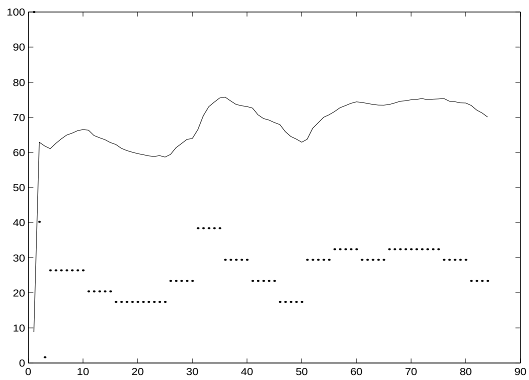

Figure 2. Identification data from the thermal process: $(\cdots)$ random-like input (electrical power) and $(-)$ output temperature (normalised values from data set $frq2$). x-axis are samples in both cases.

Since only four static points of Process I were

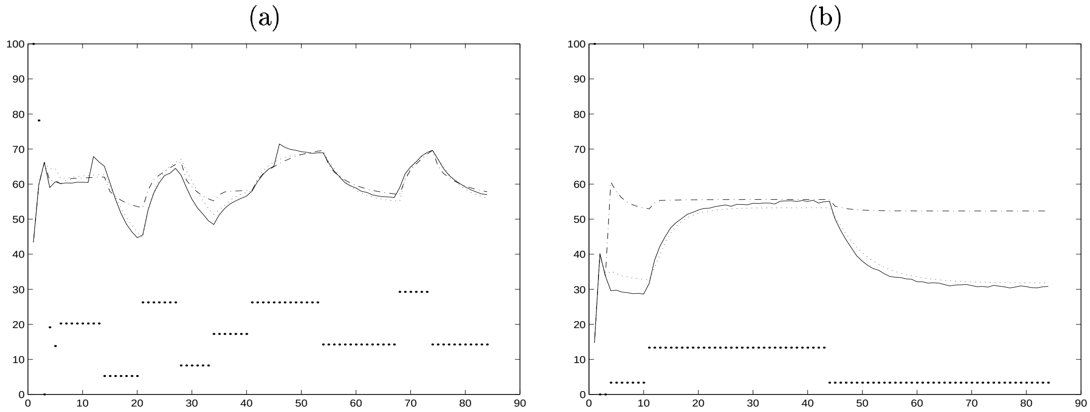

Figure 3. Free-run predictions of thermal process models. (a) Compared performances over data set $frq1$. (b) Compared performances over data set $fd1$. (●●●) Input, (–) Process I output, (···) *rbfltp1* output, and (– · –) *rbfntp1* output. x-axis is samples and y-axis is output in both cases.

obtained, from step tests realized during data collection, RBF models derived in this paper are compared with a first-order rational NARMAX model developed in (Corrêa, 2001), that performs well over all data sets. This rational model has the following static equation (Corrêa, 2001):

$$\overline{y} = \frac{0.618\overline{u}}{0.0878 + 5.5353 \times 10^{-3}\overline{u}}. \quad (4)$$

4 Experimental Results

In this section the effects of the use of linear polynomial terms in RBF network models are illustrated for the thermal process described above. The identified models, derived from the structures of Equations (1) and (3), are compared according to their ability to make long-term predictions *and* to represent fully or partially known steady-state characteristics of the system.

Two RBF models are compared in this case:

- *rbfltp1*, based in the structure of Equation (3), with $n_u = n_{u\ell} = 3$, $n_y = n_{y\ell} = 1$ and $m = 1$ (number of estimated parameters= 6),
- *rbfntp1*, based in the structure of Equation (1), with $n_u = 3$ and $n_y = 1$ for the input vector, and $m = 12$ (13 parameters),

The model order was selected as suggested in (Corrêa, 2001). This information can be very useful since for a fixed network structure the performance clearly deteriorates when the input dimension is over-specified (Sze, 1995). Otherwise, the determination of the optimal n_u and n_y values could require a tedious iterative process.

The static nonlinear function of each RBF model was obtained by solving their steady-state equations numerically[a]. In this paper, this static characteristic is not retrieved by simulation of the RBF network since it would prove to be a long and inefficient process. Furthermore, unstable operating points could not be obtained through network simulation (Hernandez and Arkun, 1991).

Models *rbfltp1* and *rbfntp1* performances as free-run predictors are shown in Figure 3, which only presents data sets $frq1$ and $fd1$. Performances over the other data sets are summarized in Table 1. Figure 4 shows the steady-state relationship between input and output for the RBF models, and also for the NARMAX rational model of Equation (4).

Table 1. Results of thermal process identification (dynamical data)

Model	max	mean	std	SSE
Random-like signal - $frq1$ data set				
rbfltp1	**2.84** $\times 10$	**3.91**	**6.98**	**3.28** $\times 10^2$
rbfntp1	7.88 $\times 10$	8.52	1.55 $\times 10$	7.16 $\times 10^2$
Identification data - $frq2$ data set				
rbfltp1	**4.45**	1.13	**1.24**	9.54 $\times 10$
rbfntp1	5.57	**1.03**	1.55	**8.62** $\times 10$
Step response - $fd1$ data set				
rbfltp1	**3.17** $\times 10$	**3.62**	**5.69**	**3.04** $\times 10^2$
rbfntp1	9.61 $\times 10^2$	2.51 $\times 10^2$	2.43 $\times 10^2$	2.11 $\times 10^4$
Step response - $fd2$ data set				
rbfltp1	5.34 $\times 10$	1.14 $\times 10$	7.80	7.62 $\times 10^2$
rbfntp1	**4.29** $\times 10$	**8.64**	**6.96**	**5.79** $\times 10^2$

Convention: all the criteria based on squared errors; std: standard deviation; SSE: sum of squared errors

An inspection of Figure 3 and Table 1 shows that *rbfltp1* is dynamically competitive over all the data sets, including for input regions not present in the identification set. Moreover, the steady-state curve presented in Figure 4 reveals that for certain input regions, the RBF network seems to be more adequate than the rational model, such as the step amplitudes of data set $fd1$ (see the asterisks in Figure 4 that shows the real stationary response of Process I, according step tests).

Also in Figure 4 it is possible to see that for input values under the identification data range, the static curves of the RBF and rational models

[a]Making $y(k) = y(k-1) = \ldots = y(k-n_y) = \overline{y}$ and $u(k) = u(k-1) = \ldots = u(k-n_u) = \overline{u}$.

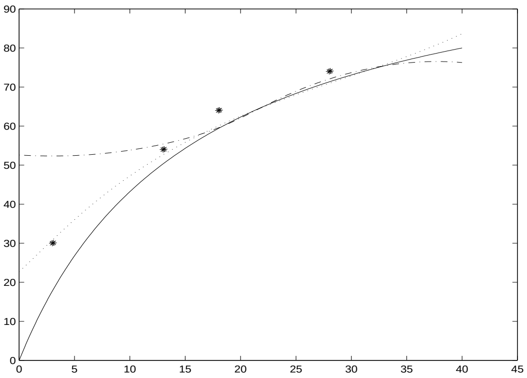

Figure 4. Static curves retrieved from thermal process models. $(\cdots)$ *rbfltp1* , $(- \cdot -)$ *rbfntp1* , $(*)$ is real response of Process I, determined from step tests and $(-)$ NARMAX rational model of Equation (4). x -axis is $\overline{u}$ (static electrical power in %) and y -axis is $\overline{y}$ (static temperature in %).

diverge from each other. However, after a more careful examination one can conclude that *rbfltp1* performance is more representative of the static characteristic for the range $3 \leq \overline{u} \leq 15$. The performance of *rbfntp1* is significantly worse for inputs outside the identification data range.

An important issue in the identification of the thermal process is training sample size. Some published results suggest that for a given training sample size, there is an optimal network structure for which a good generalization can be achieved (Sze, 1995). In general, complex network structures require an increased training sample. Hence, for the relatively short training sample length used in this work (83 data points) a small model structure seems to be a good choice. Moreover, training sample size also affects input dimension selection since the number of training data required to reach a good performance grows at an exponential rate with the dimension of the input vector (Sze, 1995). In this respect, the choice for low-order input vectors in the identification of the thermal proces could be justified again.

5 Discussion and Conclusions

The results presented in this paper show that there has been significant gain in both dynamical and steady-state performance of RBFN when linear ARX terms are included in the network topology. Improvement in the dynamical part can be easily understood by considering that the linear ARX terms form a well-known and well-established basis for dynamical systems. On the other hand, it is not so clear why the inclusion of the linear ARX terms would improve the *nonlinear* steady-state performance given that this model feature depends only on the nonlinear part of the model. At present, it is conjectured that the improved performance comes as a consequence of the addition of ARX terms because when such terms are added they start to account for the dynamics in the data thus leaving the nonlinear part of the RBFN "free" to fit the steady-state nonlinearity of the system. It seems that this scenario is somewhat analogous to the problem of parameter bias (in ARX models) induced by colored noise in the regression equation. Such bias is solved by adding moving average (MA) terms to the model, that explain the correlation present in the noise, thus enabling the ARX terms to correctly account for the linear dynamics (Aguirre, 2000). Similarly, it seems that the lack of ARX terms in RBFN models results in "bias" (revealed by a poor fit to the steady-state behavior of the system) being induced by linear dynamics in the data.

It has been shown that RBFN models with added linear polynomial terms can be used for modeling nonlinear systems, with some advantages in comparison to the traditional network topology. Based on the thermal process identification it is possible to say that the use of linear terms 1) enables the RBFN capable to perform well even with a short data set, and 2) enhances the performance in approximating the static curve especially for input regions not present in the identification set. This feature was not observed for the traditional network structure. Moreover, the inclusion of linear terms usually results in parsimonious model structures that exhibit adequate behavior.

Acknowledgements

The authors gratefully thank Rafael Lopes for the RBFN software code used in this work. GBA's research is supported by FPGA/PIC/Unileste-MG. MVC acknowledges the support of Unileste-MG, and LAA acknowledges financial support from CNPq.

References

Abreu, S. D. G. (1993). *Avaliação de desempenho de controladores auto-ajustáveis*, Dissertação de Mestrado. Programa de Pós-Graduação em Engenharia Elétrica, Universidade Federal de Minas Gerais, Belo Horizonte, MG.

Aguirre, L. A. (1997). On the structure of nonlinear polynomial models: higher-order correlation functions, spectra and term-clusters, *IEEE Transactions on Circuits and Systems - I* **5**(44): 450–453.

Aguirre, L. A. (2000). *Introdução à Identificação de Sistemas: Técnicas Lineares e Não-Lineares Aplicadas a Sistemas Reais*, UFMG, Belo Horizonte, MG.

Aguirre, L. A. and Billings, S. A. (1995). Dynamical effects of overparametrization in nonlinear models, *Physica D* **80**(1,2): 26–40.

Aguirre, L. A., Donoso-Garcia, P. F. and Santos-Filho, R. (2000). Use of *a priori* information in the identification of global nonlinear models - a case study using a buck converter, *IEEE Trans. Circuits Syst. I* **47**(7): 1081–1089.

Carlin, M. (1992). Radial basis function networks and nonlinear data modelling, *Proceedings of Neuro-Nimes'92: Neural Networks and their Applications*, Vol. EC2, France, pp. 623–633.

Chen, S., Grant, P. and Cowan, C. (1991). Orthogonal least squares learning algorithm for radial basis function networks, *IEEE Trans. Neural Networks* (2): 302–309.

Corrêa, M. V. (2001). *Identificação Caixa-Cinza de Sistemas Não-Lineares Utilizando Representações NARMAX Racionais e Polinomiais.*, Tese de Doutorado, PPGEE, Universidade Federal de Minal Gerais, Belo Horizonte, Brasil.

Henrique, H. M., Lima, E. L. and Seborg, D. E. (2000). Model structure determination in neural network models, *Chemichal Engineering Science* **55**: 5457–5469.

Jackson, I. (1988a). *Radial Basis Function Methods for Multivariate Approximation*, PhD thesis, DAMTP, Cambridge University, Cambridge, UK.

Jackson, I. (1988b). Radial basis functions: a survey and new results, *Technical report*, DAMTP, Cambridge University, Cambridge, UK. Research Report NA16.

Kadtke, J. B., Brush, J. and Holzfuss, J. (1993). Global dynamical equations and Lyapunov exponents from noisy chaotic time series, *Int. J. Bifurcation and Chaos* **3**(3): 607–616.

Knohl, T. and Unbehauen, H. (1998). Adaptive control of linear systems with unknown memoryless input nonlinearities using ANN, *The IFAC Workshop on Adaptive Systems in Control & Signal Processing*, Glasgow, Scotland, pp. 466–471.

Mao, K. Z. and Billings, S. (1997). Algorithms for minimal model structure detection in nonlinear dynamic system identification, *Int. J. Control* **68**(2): 311–330.

Mees, A. I. (1993). Parsimonious dynamical reconstruction, *Int. J. Bifurcation and Chaos* **3**(3): 669–675.

Mendes, E. M. A. M. (1995). *Identification of Nonlinear Discrete Systems with Intelligent Structure Detection*, PhD Thesis, University of Sheffield, Sheffield, UK.

Poggio, T. and Girosi, F. (1990). Networks for approximation and learning., *Proc. IEEE*, number 78, pp. 1481–1496.

Reed, R. (1993). Pruning algorithms – a survey, *IEEE Trans. Neural Networks* **4**(5): 740–747.

Rodrigues, G. G. (1996). *Identification of Real Nonlinear Systems with NARMAX Polynomial Models, (in Portuguese)*, Master's Thesis, PPGEE, Universidade Federal de Minas Gerais, Belo Horizonte, Brazil.

Spath, H. (1980). *Cluster analysis algorithms for data reduction and classification of objects*, Elis Horwood, New York.

Sze, T. L. (1995). *System Identification Using Radial Basis Function Networks*, PhD Thesis, University of Sheffield, Sheffield, UK.

Zhu, Q. M. and Billings, S. A. (1996). Fast orthogonal identification of nonlinear stochastic models and radial basis function neural networks, *Int. J. Control* **64**(5): 871–886. Reading.