UNDERACTUATED MANIPULATOR ROBOT CONTROL BY STATE FEEDBACK LINEARIZATION VIA μ-SYNTHESIS

Marco H. Terra, Tácio Luiz S. Barbeiro

Electrical Engineering Department, University of São Paulo
PO Box 359, 13560-970 São Carlos, SP, BRAZIL
E-mails: terra, tluiz@sel.eesc.sc.usp.br

Abstract— The main subject of this work is robust control via μ-Synthesis applied to one manipulator robot with 3-degrees of freedom. A set of controllers were tested in a simulator constructed to reproduce the underactuated robot dynamics and the controllers robustness was verified in one experimental underactuated robot.

Key Words— Manipulator robot; Computed Torque Method; Robust Control; μ-Synthesis.

1 Introduction

This paper focuses on robust control via a combined controller applied to one manipulator robot. The controller is designed in two steps. The first step is designed via computed torque method and the second step is a controller designed via μ-Synthesis (Zhou et al., 1996; Zhou and Doyle, 1998; Balas et al., 1998; Doyle et al., 1992). Control of underactuated manipulator (UArm) robot has been considered by several authors, see (Arai and Tachi, 1991; Arai et al., 1993; Bergerman and Xu, 1994; Terra et al., 1999) and references therein. Taking into account the uncertainties of the UArm robots, multiplicative uncertainty in the input, output disturbance, output performance, and measurement noise, the UArm controllers given in the literature, unfortunately, do not consider these points on the control designs. We have observed that performance of that controllers, considering the distance between the control design and the implementation, is not good. The procedure developed here models the uncertainties and increases the performance of the robust control. This paper is organized as follows: in the section 2 a brief presentation of underactuated dynamics is given; in the section 3 the control schema is given via feedback linearization; in the section 4 an overview on μ-synthesis is presented; in the section 5 a technique to realize μ-synthesis, known as *D-K procedure*, is given; in the section 6 the design procedure is summarized; in the section 7 the results are presented and finally the conclusions are displayed in the section 8.

2 Underactuated Manipulator Robot

In this topic will be describe an n-link, open chain, underactuated manipulator with rigid links, for more details see (Bergerman, 1996). Let q represent its joint vector and τ represent its torque vector. The dynamics equations of the manipulator are found in closed-form via the classical Lagrangian approach, as in (Craig, 1989):

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(q,\dot{q}). \quad (1)$$

In equation (1), M is the $n \times n$ symmetric, positive-definite inertia matrix, C is the $n \times n$ matrix of Coriolis and centrifugal terms, G is the $n \times 1$ vector of gravitational torques, and F is the $n \times 1$ vector of frictional torques. For convenience, are combined the vectors in the right-hand side of (1), except for $M(q)\ddot{q}$, in the vector of non-inertial torques b:

$$\tau = M(q)\ddot{q} + b(q,\dot{q}). \quad (2)$$

It is assumed that n_a degrees of freedom of the manipulator are active joints with actuators and displacement sensors, which is a typical structure of manipulators joints. The remaining $n_p(n-n_a)$ degrees of freedom are passive joints that have holding brakes instead of actuators. We will describe three different control strategies: the first is known as control strategy A, because only active joints are being controlled, and the dynamic equation (2) reduces to:

$$\tau_a = M_{aa}\ddot{q}_a + b_a \quad (3)$$

the second is known as control strategy P, because only passive joints are being controlled, and the equation (2) can be rewritten as:

$$\tau_a = \left(M_{au} - M_{aa}M_{ua}^{\#}M_{uu}\right)\ddot{q}_u + \left(-M_{aa}M_{ua}^{-1}b_u + b_a\right) \quad (4)$$

the subscripts a and u mean active and underactuated, the third is known as an AP control strategy, because both active and passive joints are being controlled. In this case, the controlled joint vector q_c will contain all elements from q_u (underactuated joints) and some elements from q_a (active joints), while q_r will contain the remaining elements from q_a not in q_c. An open-loop relationship similar to (4) can be obtained as:

$$\tau_a = \left(M_{ac} - M_{ar}M_{ur}^{\#}M_{uc}\right)\ddot{q}_c + \left(-M_{ar}M_{ur}^{-1}b_u + b_a\right). \quad (5)$$

All three control strategies above lead to open-loop relationships between $\ddot{q}_c$ and τ_a of the form:

Table 1. Relationship between controlled joints' acceleration and active torques for all three possible control strategies

Strategy	$\bar{M}_{ac}$	$\bar{b}_a$
A	M_{aa}	b_a
P	$M_{au} - M_{aa}M_{ua}^{-1}M_{uu}$	$-M_{aa}M_{ua}^{-1}b_u + b_a$
AP	$M_{ac} - M_{ar}M_{ur}^{-1}M_{uc}$	$-M_{ar}M_{ur}^{-1}b_u + b_a$

Table 2. Condition existence of $\bar{M}_{ac}^{-1}$ according to the control strategy utilized

Strategy	Existence of $\bar{M}_{ac}^{-1}$
A	always
P	M_{ua} is invertible
AP	M_{ur} is invertible

$$\tau_a = \bar{M}_{ac}\ddot{q}_c + \bar{b}_a. \quad (6)$$

We assume that dynamic coupling exists between the active and the controled joints everywhere inside the manipulator's workspace, regardless of the control strategy utilized. Consequently, controllabity is guaranteed and $\bar{M}_{ac}^{-1}$ is invertible.

3 Control Methodologies

3.1 Feedback Linearization Controller

Feedback linearization controllers have been extensively used in the control of robot manipulators, see (Lewis et al., 1993; Bergerman and Xu, 1994). The resemblance of the dynamic equation (6) with that of a fully actuated manipulator led Arai and Tachi (1991) to choose a feedback linearization controller to control the joints in q_c. The method consists in defining an auxiliary input u with:

$$\tau_a = \bar{M}_{ac}u + \bar{b}_a \quad (7)$$

such that, when $\bar{M}_{ac}$ is invertible,

$$\ddot{q}_c = u. \quad (8)$$

The feedback linearization controller (7) is to decouple and to linearize the nonlinear system (6). If the trajectory error vector, x, is defined as:

$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \quad (9)$$

with $e = q_d - q$, where q_d is the reference trajectory, we obtain the linearized error system:

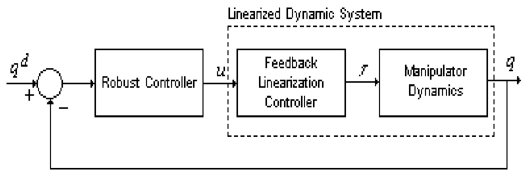

Figure 1. Underactuated Robot Control Strategy

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix}u. \quad (10)$$

This is a linear state-space system of the form:

$$\dot{x} = Ax + Bu \quad (11)$$

driven by the control input $u(t)$. The feedback linearization provides a powerful control design technique. In fact, if we select $u(t)$ so that (10) is stable, the control input torque $\tau(t)$ defined by (7) makes the robot arm move in such way that q_c will follow a desired trajectory $q_c^d(t)$. More details of this control input can be found in the next subsection. The control law consists of an inner loop feedback linearization controller, and an outer loop robust controller. A block diagram of the feedback linearization technique is present in Figure 1.

3.2 Computed Torque Controller

The problem of controlling a nonlinear system like (1) can be handled by the computed torque method (Craig, 1989). The controller can be decomposed into a model-based part and a servo part. The model-based part of the controller appears in a control law of the form:

$$\hat{\tau} = \hat{M}(q)\tau' + \hat{C}(q,\dot{q})\dot{q} + \hat{G}(q) \quad (12)$$

where $\hat{M}$ is an estimated model of the robot inertia, M. Likewise $\hat{C}$ and $\hat{G}$ are estimated models of the velocity terms and gravity terms of the actual robot.

The control torque $\tau' = \ddot{q}$ is computed by the servo part as:

$$\tau' = \ddot{q}^d + K_v(\dot{q}^d - \dot{q}) + K_p(q^d - q) \quad (13)$$

where $\{q^d, \dot{q}^d, \ddot{q}^d\}$ represents the desired trajectory and K_p and K_v are $n \times n$ diagonal matrices with each element on the diagonal being a positive gain. The closed-loop equation for the whole system is derived from Eqs. (1), (12) and (13) as:

$$\ddot{e} + K_v\dot{e} + K_pe = \hat{M}^{-1}[(M - \hat{M})\ddot{q} + (C - \hat{C})\dot{q} + (G - \hat{G})] \quad (14)$$

where $e = q^d - q$.

In a real robot, external disturbances such as friction, torque ripple of actuators, and perturbations of the payload may be a problem. If the sum of these disturbances is defined as η and added to (14), we have:

$$\ddot{e} + K_v \dot{e} + K_p e = \eta. \qquad (15)$$

This equation shows that in a real situation, model imperfections and external disturbances will introduce error and will degrade the control performance of the computed torque method.

3.3 Combined Controller

A combined controller will be utilized to compensate the undesirable effects described in the section above. The design of this controller consists of two steps. In the first step the computed torque method is used to precompensate for dynamics of the modeled plant. In the second step the μ-controller is used to postcompensate for the residual error which is not completely removed by the computed torque method. Thus, the combined controller is capable of performing robust tracking control. A schematic diagram of the combined controller is presented in the Figure 2. The linearization provided by the computed torque method is useful in the design procedure. A state-space realization is derived from the equation (15) and can be written as:

$$\begin{bmatrix} \dot{e} \\ \ddot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} u \end{bmatrix}. \qquad (16)$$

This is a linear state-space system that represents the nominal plant model of the system.

4 Overview on μ-Synthesis

4.1 System Description

Consider the system displayed in the Figure 3. The system labeled P is the open-loop interconnection and contains all of the known elements including the nominal plant model, performance and uncertainty weighting functions. The Δ block is the uncertain element, which parametrizes all of the assumed model uncertainty in the problem. The controller is K. Three sets of inputs enter P: perturbation inputs w, disturbances d, and controls u. Three sets of outputs are generated: peturbation outputs z, errors e, and measurements y.

4.2 The μ-Problem

Given the system, for a robust performance, G is chosen as:

$$G = P = \left[\begin{array}{cc|c} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ \hline P_{31} & P_{32} & P_{33} \end{array}\right]$$

and a simple linear fractional transformation may be written involving the controller and the plant:

$$M = \mathcal{F}_l(G,K) = \\ = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}. \qquad (17)$$

For some transfer matrix M we have the following synthesis problem:

$$\min_K \|\mathcal{F}_l(G,K)\|_\mu \qquad (18)$$

which is subject to the internal stability of the nominal model control. Synthesis μ may be obtained by scaling and applying $\|.\|_\infty$. A reasonable approach is to solve:

$$\min_K \inf_{D,D^{-1}\in\mathcal{H}_\infty} \|D\mathcal{F}_l(G,K)D^{-1}\|_\infty \qquad (19)$$

by iteratively solving for K and D. This procedure is called *D-K iteration*. The scaling matrix $D(s)$ can be stable and minimum phase. It is chosen such that $D(s)\Delta(s) = \Delta(s)D(s)$. For a fixed scaling transfer matrix D,

$$min_K \|D\mathcal{F}_l(G,K)D^{-1}\|_\infty \qquad (20)$$

is a standard H_∞-optimization problem. For a given stabilizing controller K,

$$inf_{D,D^{-1}\in\mathcal{H}_\infty} \|D\mathcal{F}_l(G,K)D^{-1}\|_\infty \qquad (21)$$

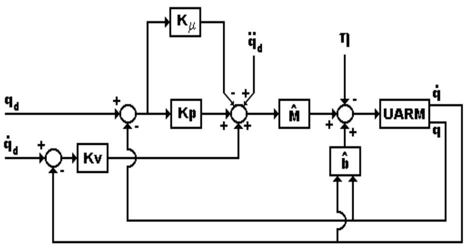

Figure 2. Mixed μ-Synthesis and Computed Torque Control Strategy

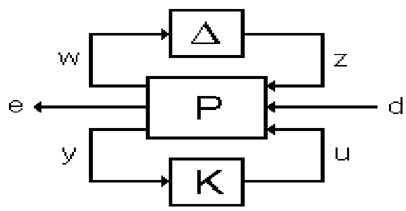

Figure 3. LFT Description of Control Problem

is a standard convex optimization problem and it can be solved pointwise in the frequency domain:

$$\sup_{w} \inf_{D_w \in \mathcal{D}} \overline{\sigma}[D_w \mathcal{F}_l(G,K)(jw) D_w^{-1}]. \quad (22)$$

Indeed,

$$\begin{aligned} &\inf_{D,D^{-1}\in\mathcal{H}_\infty} \|D\mathcal{F}_l(G,K)D^{-1}\|_\infty = \\ &= \sup_w \inf_{D_w\in\mathcal{D}} \overline{\sigma}[D_w \mathcal{F}_l(G,K)(jw) D_w^{-1}]. \quad (23) \end{aligned}$$

There is always a rational function $D(s)$ uniformly approximating the magnitude frequency response D_w. Considering the subset of $\mathbb{C}^{n\times n}$:

$$\mathcal{D} = \begin{bmatrix} diag[D_1,...,D_S, d_1 I_{m1},...,d_{F-1} I_{mF-1}, I_{mF}] : \\ D_i \in \mathbb{C}^{r_i\times r_i}, D_i = D_i^* > 0, d_j \in \mathbb{R}, d_j > 0 \end{bmatrix}$$

when $S = 0$,

$$D_w = diag(d_1^w I, ..., d_{F-1}^w I, I) \in \mathcal{D} \quad (24)$$

which is a block-diagonal scaling matrix applied pointwise across frequency to the frequency response $\mathcal{F}_l(G,K)(jw)$. For more details see (Zhou et al., 1996).

5 D-K Procedure

The D-K Iterations may be summarized in the following steps:

(i) Fix an initial estimate of the scaling matrix $D_w \in \mathcal{D}$ pointwise across frequency;

(ii) Find scalar transfer functions $d_i(s), d_i^{-1}(s) \in \mathcal{RH}_\infty$ for $i = 1,2,...,(F-1)$ such that $|d_i(jw)| \approx d_i^w$. This step can be done using the interpolation theory (Youla and Saito, 1967).

(iii) Let,

$$D(s) = diag(d_1(s)I, ..., d_{F-1}(s)I, I).$$

Construct a state space model for system in the Figure 4:

$$\hat{G}(s) = \begin{bmatrix} D(s) & \\ & I \end{bmatrix} G(s) \begin{bmatrix} D^{-1}(s) & \\ & I \end{bmatrix};$$

(iv) Solve an H_∞-optimization problem to minimize:

$$\|\mathcal{F}_l(\hat{G},K)\|_\infty$$

over all stabilizing $K's$. Note that this optimization problem uses the scaled version of G. Let its minimizing controller be denoted by $\hat{K}$;

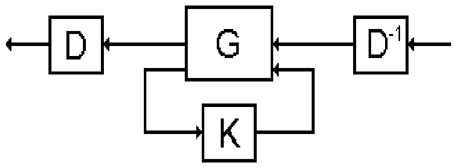

Figure 4. μ-Synthesis via Scaling

(v) Minimize $\overline{\sigma}[D_w \mathcal{F}_l(G,\hat{K}) D_w^{-1}]$ over D_w, pointwise across frequency. Note that this evaluation uses the minimizing $\hat{K}$ from the last step, but that G is unscaled. The minimization itself produces a new scaling function. Let this new function be denoted by $\hat{D}_w$;

(vi) Compare $\hat{D}_w$ with the previous estimate D_w. Stop if they are close, but, otherwise replace D_w with $\hat{D}_w$ and return to step (ii).

6 Design Procedure

Mutools MATLAB toolbox was utilized to design the controllers. The interconnection structure presented in the Figure 5, including the nominal plant model, performance and uncertainty weighting functions, must be defined to a corrrect design procedure. The state-space realization for the nominal plant was presented in the section 3 and the procedure to determine the weighting functions can be found in (Balas et al., 1998; Doyle et al., 1992; Zhou and Doyle, 1998). The performance objectives are related to the frequency response of the sensitivity function (S). Calculating the natural frequency w_n, the damping ratio ϵ, the bandwith ω_b and the peak sensitivity M_s, the following performance weighting function can be determined:

$$W_p = diag\{F_p, F_p, F_p\}$$

$$F_p = \frac{\frac{s}{M_s} + \omega_b}{s + \omega_b \epsilon} \quad (25)$$

Calculating the maximum gain M_u of KS and the controller bandwith ω_{bc}, the following control weighting function W_{del} can be selected:

$$W_{del} = diag\{F_{del}, F_{del}, F_{del}\}$$

$$F_{del} = \frac{s + \frac{\omega_{bc}}{M_u}}{\epsilon_1 s + \omega_{bc}} \quad (26)$$

7 Results

The controllers were tested in a simulator constructed to reproduce the underactuated robot dynamics and in the experimental robot UArmII,

Table 3. Weighting Functions Parameters

$M_s = 1$	$M_u = 15$
$\omega_b = 1rad/s$	$\omega_{bc} = 10000rad/s$
$\epsilon = 0.0001$	$\epsilon_1 = 0.0001$

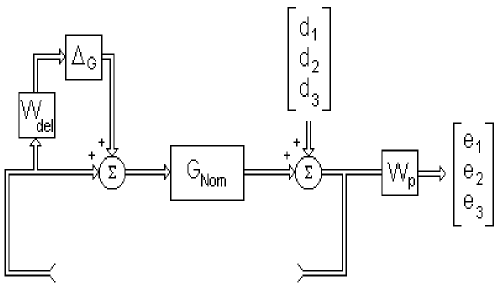

Figure 5. Open Loop Interconnection Structure

see (Terra et al., 1999) for more details. The parameters utilized for determination of M and b are displayed in Table 4. The validity of the control method proposed and the controller's robustness can be verified in the experimental results, see Figures 6, 7 and 8 for AAA configuration and Figures 12, 13 and 14 for APA configuration.

8 Conclusions

In this paper was available the underactuated robot control via μ-Synthesis. The computed torque technique provides a good tool to determine state-space realizations for a underactuated manipulator, required in the design procedure. The experimental results presented in the item 7 show the robustness of this controller with an interesting performance in the presence of parametric uncertainties.

Acknowledgment

We would like to express our gratitude to FAPESP (Undergrant $N°99/01493-1$ and $N°98/00649-5$), for the financial support.

Table 4. Robot Parameters

Link	$m_i(Kg)$	$I_i(Kgm^2)$	$l_i(m)$	$l_{ci}(m)$
1	0.850	0.0075	0.203	0.096
2	0.850	0.0075	0.203	0.096
3	0.625	0.0060	0.203	0.077

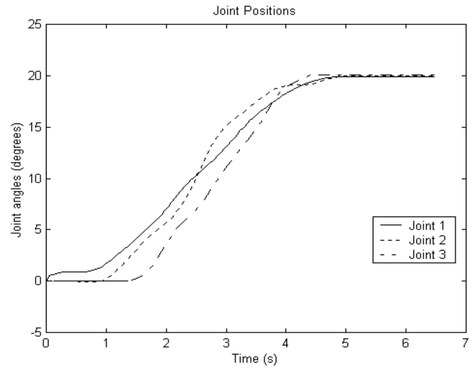

Figure 6. Joint Positions (Experimental) - Conf. AAA

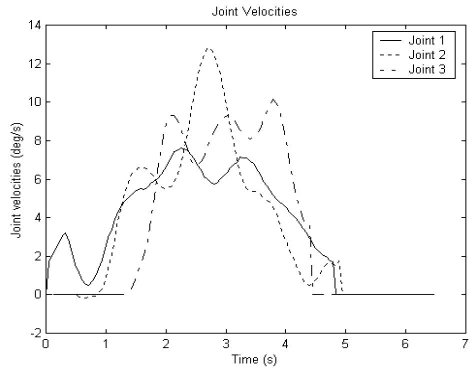

Figure 7. Joint Velocities (Experimental) - Conf. AAA

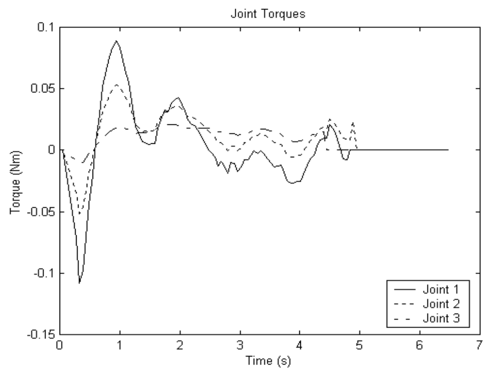

Figure 8. Joint Torques (Experimental) - Conf. AAA

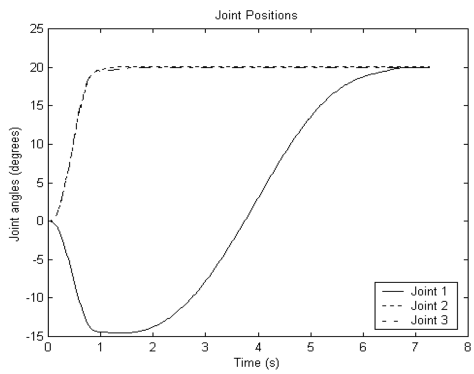

Figure 9. Joint Positions (Simulation) - Conf. APA

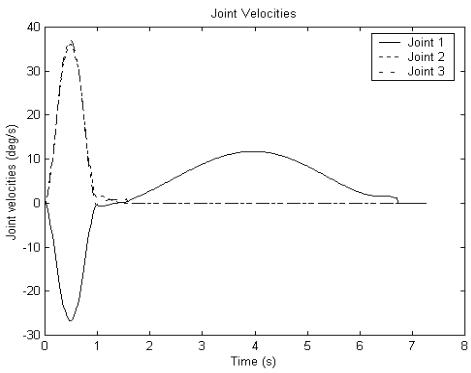

Figure 10. Joint Velocities (Simulation) - Conf. APA

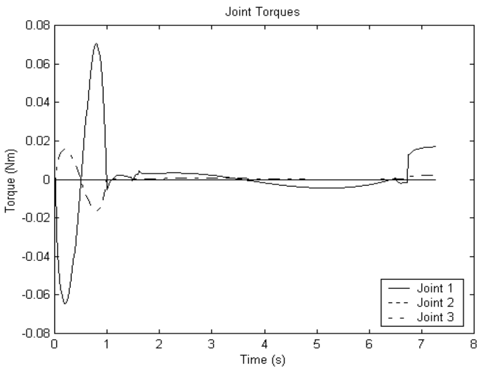

Figure 11. Joint Torques (Simulation) - Conf. APA

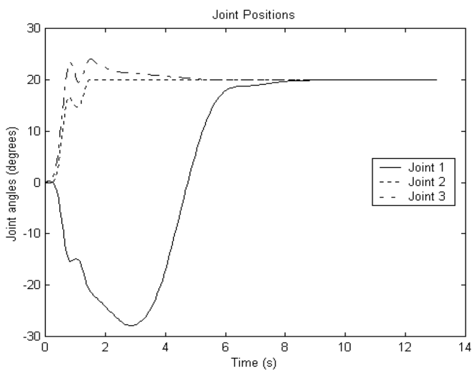

Figure 12. Joint Positions (Experimental) - Conf. APA

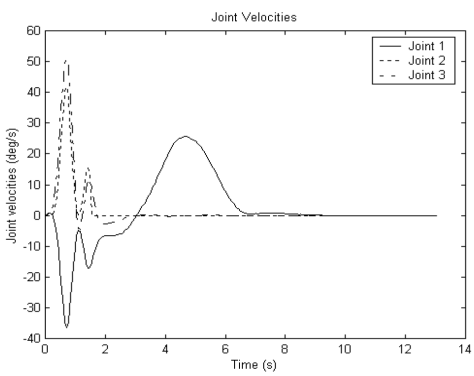

Figure 13. Joint Velocities (Experimental) - Conf. APA

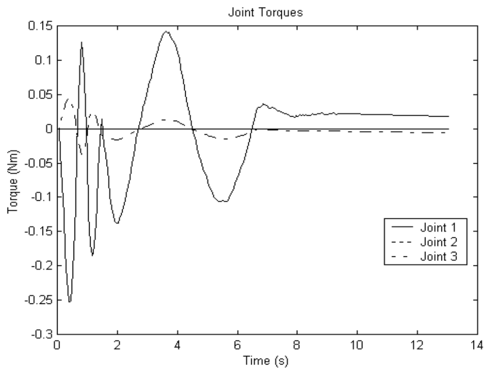

Figure 14. Joint Torques (Experimental) - Conf. APA

References

Arai, H. and Tachi, S. (1991). Position control of a manipulator with passive joints using dynamic coupling, *IEEE Transactions on Robotics and Automation* **7**(4): 528–534.

Arai, H., Tanie, K. and Tachi, S. (1993). Dynamic control of a manipulator with passive joints in operaton space, *IEEE Transactions on Robotics and Automation* **9**(1): 85–93.

Balas, G., Doyle, J., Glover, K., Packard, A. and Smith, R. (1998). *μ-Analisis and Synthesis Toolbox - User's Guide*, MathWorks.

Bergerman, M. (1996). *Dynamics and control of underactuated manipulators*, PhD. Thesis. Carnegie Mellon University, USA.

Bergerman, M. and Xu, Y. (1994). Robust control of underactuated manipulators: analysis and implementation, *IEEE International Conference on Systems, Man and Cybernetics*, pp. 925–930.

Craig, J. J. (1989). *Introduction to robotics mechanics and control*, Addison-Wesley.

Doyle, J., Francis, B. and Tannenbaum, A. (1992). *Feedback control theory*, Macmillan.

Lewis, F., Abdallah, C. and Dawson, D. (1993). *Control of robot manipulators*, Macmillan.

Terra, M., Siqueira, A. and Bergerman, M. (1999). Underactuated manipulator robot control via linear matrix inequalities, *IEEE International Conference on Decision and Control.*

Youla, D. and Saito, M. (1967). Interpolation with positive-real functions, *Journal of The Franklin Institute* **284**(2): 77–108.

Zhou, K. and Doyle, J. (1998). *Essentials of robust control*, Pretince-Hall.

Zhou, K., Doyle, J. and Glover, K. (1996). *Robust and optimal control*, Pretince-Hall.