A STUDY OF ANTI-WINDUP TECHNIQUES APPLIED TO THE CONTROL OF ROBOT MANIPULATORS

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Abstract— In this paper we analyze the behavior of two saturation compensation (anti-windup) techniques in the control of robot manipulators. The standard observer-based anti-windup (Åström and Rundqwist, 1989) is studied together with the technique proposed in (Reginatto and de Pieri, 2000) based on the unification of local and global controllers. Simulation results are presented for a 2-link planar robot manipulator illustrating the performance of each technique in certain robot tasks. Qualitative conclusions are drawn on the base of these simulation results.

Key Words— Robot manipulators, saturation compensation, anti-windup, observer-based anti-windup

1 Introduction

A basic problem in controlling robots is to ensure that the manipulator follows a planned desired trajectory or moves through a free space to a specified point. In performing such tasks, the torque limits of the actuators may be reached, thus leading to the windup problem, especially due to the fact that robot controllers often include integral action (Lewis et al., 1993; Spong and Vidyasagar, 1989; Qu and Dawson, 1996).

Virtually, every actuator has limited capacity, so that the actuator saturation problem is inherent to almost all actual systems. Of course, the effect of actuator saturation may be less important in many systems specially when they operate close the equilibrium points so that actuators do not reach their limits. This fact has motivated engineers and practitioners over the time to ignore the actuator limits in the design of compensators. Such procedure, however, may lead to difficulties in the actual system which will not behave as desired due to the effect of actuator limits. An approach that has been pursued in the literature to address this problem is the anti-windup scheme.

Anti-windup schemes have been considerably studied over the last decades specially in the context of linear systems with linear controllers. Among these schemes, the observer-based anti-windup (Åström and Rundqwist, 1989; Åström and Hägglund, 1988) has gained recognition by its design simplicity and intuitive behavior. More general approaches to the anti-windup problems have appeared more recently (Kothare and Morari, 1997; Teel and Kapoor, 1997a; Kapoor et al., 1998; Barbu et al., 2000). In the context of robot manipulators, a strategy for saturation compensation has been reported in (Reginatto and de Pieri, 2000). The strategy is based on the unification of local and global controller procedure of (Teel and Kapoor, 1997b), which allows to consider nonlinear systems.

In this paper we wish to analyze the effectiveness of these anti-windup techniques in the control of robot manipulators. We chose the standard observer-based anti-windup technique (Åström and Rundqwist, 1989; Åström and Hägglund, 1988), and the saturation compensation strategy of (Reginatto and de Pieri, 2000). Our comparison is based on simulation results and allows to draw qualitative conclusions about the performance and weaknesses of each technique.

The paper is organized as follows. In section 2 we pose the problem and clarify the issue we want to address. Section 3 present the development of the control methodology and the anti-windup/ saturation compensations techniques we wish to study. In section 4 we apply these techniques to the control of a 2-link planar robot manipulator. We present simulation results to show the transient response obtained with the different control strategies We end the paper with a discussion comparing several aspects of the anti-windup/saturation compensation schemes.

2 Control of robot manipulators

We consider a 2-link robot manipulator described by the differential equation

$$M(q)\,\ddot{q} + C(q,\,\dot{q})\dot{q} + G(q) = \tilde{\tau} - \tau_d \qquad (1)$$

where $q = [\theta_1,\, \theta_2]^T$ is the vector of joint angles, $\tilde{\tau}$ and τ_d are joint torque and load torque respectively, M is the inertia matrix, C accounts for centrifugal and Coriolis terms, and G accounts for gravity terms. We also denote $V(q,\dot{q}) := C(q,\,\dot{q})\dot{q}$.

Standard control strategies for robot manipulators consist in two nested loops, the inner one being a non-linear static state feedback providing either a feedback linearization or feedforward gravity compensation, the outer loop being a standard PD or PID controller (Lewis et al., 1993; Rocco, 1996) to provide regulation/tracking.

In the feedback linearization procedure the torque is computed as $\tilde{\tau} = \tau_c$, where

$$\tau_c := M(q)u + V(q,\,\dot{q}) + G(q) \tag{2}$$

This choice renders the robot dynamics linear from u to q, i.e,.

$$\ddot{q} = u - M^{-1}(q)\,\tau_d \tag{3}$$

On the other hand, gravity compensation consists in directly compensating for the gravity term $G(q)$, while not canceling the remaining robot nonlinearities. The gravity compensation is implemented as $\tilde{\tau} = \tau_g$, where

$$\tau_g := u + G(q) \tag{4}$$

The closed-loop dynamics with gravity compensation (1), (4) results in

$$M(q)\,\ddot{q} + V(q,\,\dot{q}) = u - \tau_d \tag{5}$$

The outer loop is often a PD or PID controller. Integral action, PID structure, is often employed to avoid steady-state errors in the presence of load torque or other small disturbances/uncertainties. In general, we represent this controller as follows

$$\dot{x}_c = A_c x_c + B_c q + E_c q_d \tag{6}$$
$$u = C_c x_c + D_c q + F_c q_d \tag{7}$$

where q_d stands for a desired reference trajectory.

We consider that each joint i is driven by an actuator with delivered torque limited to $-M_i$ and M_i. We model such constraints as saturation functions in the robot torque inputs, i.e., $\tilde{\tau} = \mathrm{sat}(\tau)$ where $\mathrm{sat}(\tau) = [\mathrm{sat}(\tau_1),\,\mathrm{sat}(\tau_2)]^T$, and

$$\mathrm{sat}(\tau_i) = \max\{-M_i,\ \min\{M_i,\ \tau_i\}\} \tag{8}$$

The actual implementation of the control laws (2) and (4) would then become,

$$\tau = \tau_c := M(q)u + V(q,\,\dot{q}) + G(q) \tag{9}$$
$$\tau = \tau_g := u + G(q) \tag{10}$$

The design of these controller are often performed by neglecting the actuator limits, and will be referred to as *nominal designs*. Since actual actuators do have limits, performance degradation and even stability problems are expected to happen in the actual closed-loop system. This problem (known as the is the ***windup*** problem) is specially important in the case the robot arm is subjected to fast and large transient. The ***anti-windup*** problem (or, more generally, saturation compensation problem) then consist in introducing additional control actions intended to counteract the effects of actuator saturation. In this paper we analyze the behavior of two anti-windup/saturation compensation schemes in the control of robot manipulators.

3 Saturation compensation/Anti-windup strategies

3.1 Observer based anti-windup

The classical observer based anti-windup technique (Åström and Rundqwist, 1989; Åström and Hägglund, 1988), consists in directly modifying the nominal controller by means of a feedback term that accounts for the amount of actuator saturation. This technique, as it has been proposed, assumes the nominal controller and the plant to be linear. In spite of that, its application to robot manipulators has been motivated by the fact that the nominal controller is a PID and the robot dynamics can be linearized by state feedback.

The observer based anti-windup modifies the controller (6)-(7) by adding the feedback term $L(\mathrm{sat}(u) - u)$ as follows

$$\dot{x}_c = A_c x_c + B_c q + E_c q_d + L(\mathrm{sat}(u) - u) \tag{11}$$

The denomination of this technique is motivate by the observer structure of (11). The state x_c is being modified in the attempt to reproduce $\mathrm{sat}(u)$.

The design of the observer-based anti-windup consists in finding the matrix gain L. In the setting of robot manipulators, this task can be performed on the basis of the double integrator model obtained after a feedback linearization.

One approach for designing L is simply the placement of the eigenvalues of $A_c - LC_c$. Assuming observability of the pair (C_c, A_c) this is always possible. This design, however does not give precise stability guarantees for the closed-loop system.

A design with global stability guarantees for the saturated linear system is given in (Kapoor et al., 1998). It depends on special passivity properties of the linear plant. When not satisfied, local stability can, in general, be guaranteed.

In (Saeki and Wada, 2000) an LMI-based design is proposed, employing the small gain theorem as a design criterion. Local stability is guaranteed, but no clear statement is given regarding the domain of attraction.

Remark 1 *It must be remarked that although the design of the observer-based anti-windup may* *guarantee some stability properties for the linear robot model, the same properties do not hold in general for the actual robot. This is so because the feedback linearization is also part of the control signal. As a result, it is not true that we can consider the robot model plus feedback linearization as a linear saturated system.*

3.2 Local-global technique

In this approach, the saturation compensation is achieved by "mixing" the control signal generated by the nominal controller with another control action determined by a controller specially designed to deal with the stabilization problem under saturation, called the *global controller*. The underlining idea is that, the nominal controller would meet performance specifications in the absence of saturation while the global controller would met stability requirements in the presence of saturation. The mixing of these two actions would lead to a system with "in the large" stability guarantees and "local" performance guarantees. We will refer to this strategy as the *local-global* technique.

To develop the saturation compensation scheme, let us rewrite the robot model in state space coordinates $x_p := [q^T, \dot{q}^T]^T$, neglecting the load torque

$$\dot{x}_p = f(x_p, \operatorname{sat}(\tau)) \tag{12}$$

where

$$f(x_p, \bar{\tau}) := \begin{bmatrix} x_{p3} \\ x_{p4} \\ M^{-1}(x_p)\,[\bar{\tau} - V(x_p) - G(x_p)] \end{bmatrix}$$

Define also the auxiliary system, which consists in the robot manipulator without actuator constrains, as follows,

$$\dot{z} = f(z, \tau) \tag{13}$$

We design the nominal controller on the basis of system (13). Let q_d be the desired joint angle trajectory let this controller be given by

$$\begin{aligned} \dot{x}_c &= g(x_c, u_c, q_d) \\ y_c &= k(x_c, u_c, q_d) \end{aligned} \tag{14}$$

Assume that (14) has been designed to provide a given desired performance when in closed loop with (13), i.e., $u_c = z$ and $\tau = y_c$. Notice that the nominal controller can be nonlinear in this case, so that the actions of gravity compensation or even feedback linearization can be handled properly. We assume that (14) already includes all these actions.

Saturation compensation is achieved by introducing a dynamical compensator given in terms of a function ρ, which accounts for the action of the global controller (to be specified later in this section). Its structure is as follows

$$\begin{aligned} \dot{\xi} &= f(x_p, \operatorname{sat}(y_c + v_1)) - f(x_p + v_2, y_c) \\ v_1 &= \rho(x_p, x_p + v_2) \\ v_2 &= -\xi \end{aligned} \tag{15}$$

This compensator is connected in closed-loop with (12) and (14) according to

$$u_c = x_p + v_2, \quad \tau = y_c + v_1 \tag{16}$$

The guidelines for the design of function ρ in (15) and the properties of the closed-loop system obtained with the addition of the saturation compensator (15) are summarized in the next theorem, which is a consequence of the result in (Teel and Kapoor, 1997b). A proof of this result can be found in (Reginatto and de Pieri, 2000).

Theorem 1 *Consider system (12) in closed-loop with (14) and (15) according to (16). Let $(z(t), \tilde{x}_c(t))$ and $\tilde{y}_c(t)$ denote a trajectory for the system (13) in closed-loop with (14) and let $(z^*, \tilde{x}_c^*)$ and $\tilde{y}_c^*$ denote its steady-state value. Assume ρ is such that*

1. *$\rho(x, x) = 0$, for all x in a neighborhood of the origin*
2. *$x = z^*$ is a locally asymptotically stable equilibrium point for the system*

$$\dot{x} = f(x, \operatorname{sat}(\tilde{y}_c^* + \rho(x, z^*))) \tag{17}$$

Then, for the closed-loop system (12), (14), (15), (16), it holds that

1. *If $\operatorname{sat}(\tilde{y}_c(t)) = \tilde{y}_c(t)$, $\forall t \geq 0$ and $\xi(0) = 0$, then $x_p(t) = z(t)$, for all $t \geq 0$.*
2. *$(x_p, x_c, \xi) = (z^*, \tilde{x}_c^*, 0)$ is a locally asymptotically stable equilibrium point.*

The first statement in Theorem 1 ensures that the saturation compensation scheme (15) preserves the local performance induced by the controller (14) when the actuator limits are not exceeded. The second, guarantees local asymptotic stability of the closed-loop system and convergence to the same equilibrium point as the unsaturated system would converge to.

The conditions on the function ρ in Theorem 1 require it to be a static state feedback guaranteeing asymptotic stability for then system (12). Although the design of ρ entails the saturated system, it does not involve any performance requirements, thus being much simpler than a direct stability and performance design for the system (12). Moreover, it is completely independent of the controller (14), thus allowing for any appropriate controller design, on the basis of the unsaturated system, to yield the desired local performance.

4 Application to a 2-link robot manipulator

We consider a 2-link planar robot manipulator whose parameters are given by $m_1 = 1.5$Kg, $m_2 = 1$Kg, $r_1 = 1.2$m, and $r_2 = 1$m. The gravity acceleration is taken as $9.8m/s^2$. The actuators at joints 1 and 2 are supposed to deliver a maximum torque of $50Nm$ and $20Nm$, respectively.

In our study, we are most concerned with point-to-point tasks, i.e., the motion of the robot arms from one given point to another either in the workspace or in the joint space. For this tasks, we consider independent joint control with a PID controller with gravity compensation. Besides the usual PID structure, we also consider this modified one, which is more adequate for positioning the arms without overshoot,

$$\begin{bmatrix} \dot{x}^i_{c1} \\ \dot{x}^i_{c2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1/\tau \end{bmatrix} \begin{bmatrix} x^i_{c1} \\ x^i_{c2} \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q^i_d \\ q^i \end{bmatrix} \quad (18)$$

$$u^i = \begin{bmatrix} k_i, \; k_d/\tau^2 \end{bmatrix} \begin{bmatrix} x^i_{c1} \\ x^i_{c2} \end{bmatrix} + \begin{bmatrix} 0, \; -k_p - k_d/\tau \end{bmatrix} \begin{bmatrix} q^i_d \\ q^i \end{bmatrix} \quad (19)$$

where k_p, k_d, and k_i are non-negative design parameters, and the superscript i indicates the joint number. Our nominal controller then consists in the combination of (18)-(19) with

$$\tau^i := u^i + G(q) \quad (20)$$

The tuning of the parameters k_p, k_d and k_i has been performed by eigenvalue assignment on the basis of the liner model obtained from feedback linearization, which lead us to the following parameters: $k_p = 860$, $k_d = 48$, and $k_i = 3580$. Then, these gains have been multiplied by 5 to counteract the effect of the nonlinearities in controller (20). The same gains have been used for both joints.

The observer-based anti-windup has been designed by means of a placement of the eigenvalues of the matrix $A_c - LC_c$. For this design, we have chosen eigenvalues that induce a faster dynamics than the robot dynamics in closed-loop. For the simulation results, we have chosen L so that to place the eigenvalues of $A_c - LC_c$ at -50 and -100 in both joints. A worse behavior has been observed for values close to the $j\omega$ axis.

The function ρ that plays the role of a global controller in the local-global scheme have been designed on the base of a PD controller with gravity compensation and is given by

$$\rho(x, y) = [K_1, \; K_2]\,(y - x) + G(x) - G(y) \quad (21)$$

The controller (21) is well known to globally stabilize the unsaturated system (13) for constant reference signals and zero load torque (Lewis et al., 1993, Chap. 3). Thus, it is also a locally stabilizer for the saturated system (12). The tuning of the parameters has also been performed with eigenvalue assignment on the basis of the linear dynamics (double integrator) of the robot manipulator. We have obtained the gains: $K_1 = 200$ and $K_2 = 80$. We have used the same gains for both joints.

4.1 Simulation results

We first consider reference step changes in the joint angles. With the given design, the unsaturated response for this reference signal converges exponentially to the reference value without overshoot. Moreover, the coupling between the joint is negligible, i.e., the influence of the motion of one joint has almost no influence on the other.

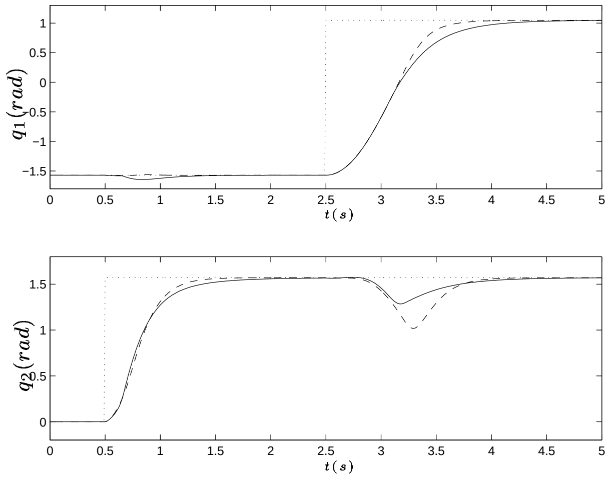

Figure 1. Robot manipulator with controller (20). Joint angle responses: dotted - reference; dashed - observer-based anti-windup; solid - local-global saturation compensation.

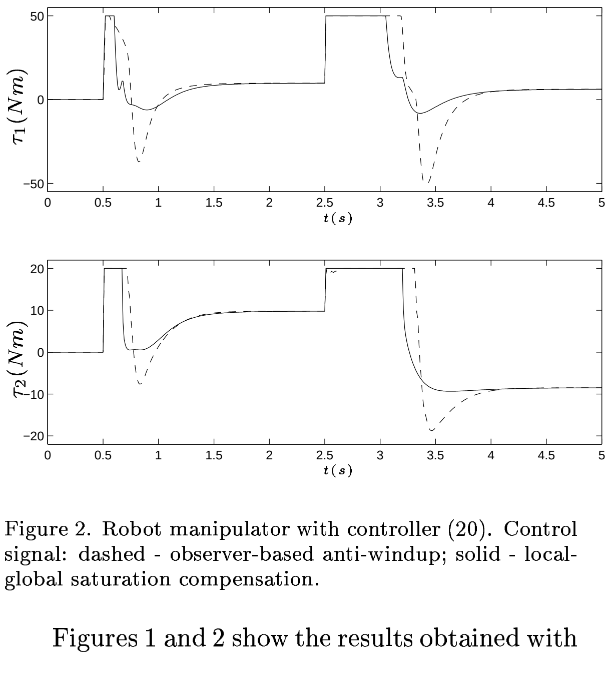

Figure 2. Robot manipulator with controller (20). Control signal: dashed - observer-based anti-windup; solid - local-global saturation compensation.

Figures 1 and 2 show the results obtained with the nominal controller (20) in the presence of saturation. The two techniques, observer-based anti-windup and local-global saturation compensation, produce similar transient responses, showing small or none overshoot. It can be observed a coupling effect between the joints. This is due to the control saturation and is more evident in the observer-based anti-windup case.

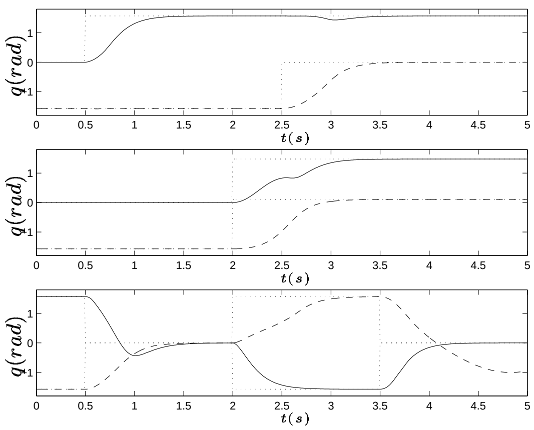

Figure 3. Robot response with observer-based anti-windup: solid - joint 1; dashed - joint 2; dotted - reference

Figures 3 and 4 show the transient response to different reference signals for both schemes: the observer-based anti-windup and local-global saturation compensation. It can be verified that the local-global transient response seems less sensitive to the change in the reference signal, showing smaller overshoot and less coupling effects between the joints compared to the observer-based anti-windup.

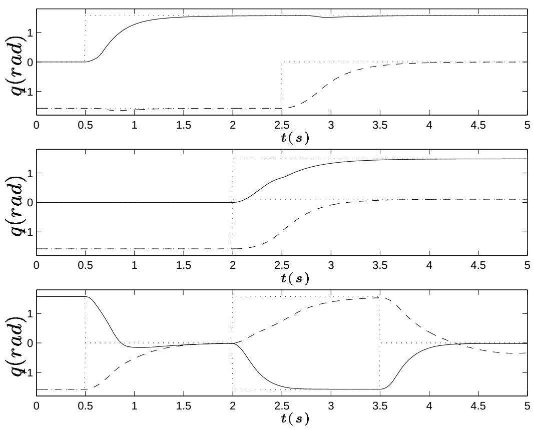

Figure 4. Robot response local-global saturation compensation: solid - joint 1; dashed - joint 2; dotted - reference

For Figures 5 and 6 we have employed a standard PID structure and considered a soft reference change (3rd order polynomial approximation) between 2 points in the joint space. Two different PID settings have been tested with all other control parameters equal.

$$k_p = 2000, \; k_i = 200, \; k_d = 180 \qquad (22)$$

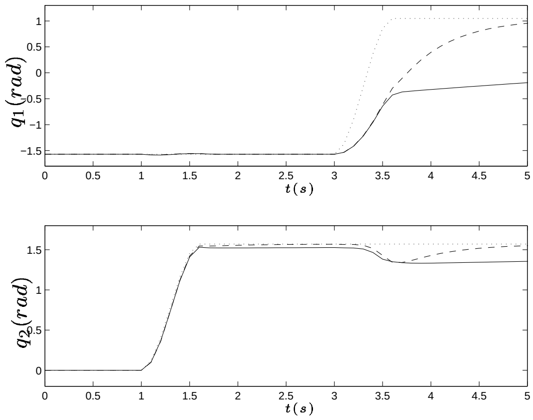

Figure 5. Robot response with controller (20) and observer-based anti-windup: solid - PID setting (22); dashed - PID setting (23); dotted - reference.

$$k_p = 2596, \; k_i = 4400, \; k_d = 194 \qquad (23)$$

Even though both setting yield to very similar results for the unsaturated system, considerably different results are obtained for the observer-based anti-windup. The result shows that the observer-based anti-windup is sensitive the nominal controller setting, as opposed to the local-global technique.

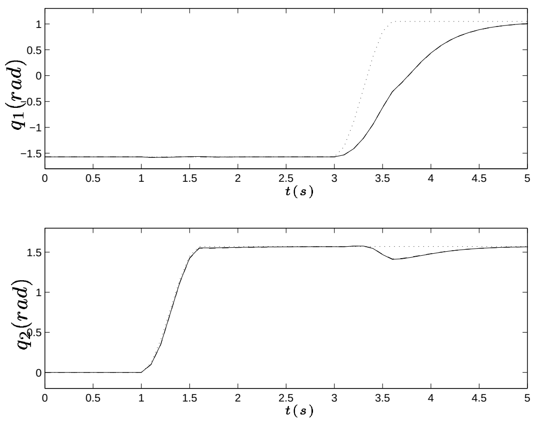

Figure 6. Robot response with controller (20) and local-global saturation compensation: solid - PID setting (22); dashed - PID setting (23); dotted - reference.

5 Discussion

The observer-based anti-windup is clearly a simpler design than the local-global saturation compensation. This is the main motivation to employ this technique to compensate for saturation in robot manipulators. On the other hand, this technique was not conceived for nonlinear systems, thus possibly leading to some problems. On the other hand, local-global saturation compensation introduces more flexibility to the design, allowing to employ nominal controllers other than classical PID's. It then opens a wide spectrum of possible design for robot manipulators that account for saturation. Next, we establish additional comparative points in order to clarify the advantages/disadvantages of each technique.

Transient performance. For a *given specific task*, it is probably possible to find tuning parameters that yield very similar transient responses for both observer-based anti-windup and local-global saturation compensation. It has been observed, however, that the transient response for the local-global scheme is less dependent on the input magnitude than in the observer-based anti-windup case. Moreover, it is also less dependent on the particular setting of the nominal controller, i.e., the observer-based anti-windup may yield considerably different transient responses for PID settings that give similar unsaturated responses.

Stability. It is difficult to provide stability guarantees with the observer-based anti-windup. The major difficulty is that it is a design intended for linear saturated systems. The presence of saturation does not allow the use of feedback linearization to render the system dynamics linear, so the robot manipulator cannot be considered as a linear saturated system. One possible approach is the Jacobian linearization, when the operating region of the manipulator is small.

Stability in the local-global scheme is most concerned with the design of the global controller. The stability properties guaranteed by the global controller are, let's say, "inherited" by the closed-loop system. Since the design encompasses nonlinearities, it is possible to perform a design with closed-loop stability guarantees in the large, i.e., taking into account the nonlinear characteristic of the manipulator.

Design flexibility. The local-global technique allows more design flexibilities since the designs of the local and global controllers are independent. Moreover, the local controller can also be nonlinear, thus allowing to properly deal with nonlinear systems. Of course, the closed-loop system performance is affected by the interaction of the local (nominal) and global controllers. However, according to the results, this interaction is considerably more evident in the observer-based anti-windup.

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