AN INSTRUMENTAL VARIABLE ALGORITHM FOR CLOSED LOOP IDENTIFICATION

GINALBER L. O. SERRA, JOSÉ TARCÍSIO C. FILHO

Control and Automation Laboratory, Electrical Engeneering Department, Federal Uniersity of Maranhão
65080-027 – São Luís – MA – Brazil
phone : 55 (098)217-8225 fax : 55 (098) 217-8241
E-mails:ginalber@dee.ufma.br, *tarcisio*@dee.ufma.br

Abstract— The most controller design schemes were developed based on the dynamic behavior of the plant by a priori model, which is not always valid, because the model used for the controller design may not well represent the dynamic behavior of the plant in closed loop. Thus, the performance of the control system may seriously degrade. Therefore, it is need iterative controller design schemes, consisting of closed loop plant model identification and controller redesign based on identified model cycles. This paper present an adaptive weighted instrumental variable (WIV) algorithm for system identification based on the numerically robust orthogonal Householder transformations. The selection and use of chosen instruments shows acceptable results: the instruments present desirable statistical properties resulting in an asymptotically unbiased estimate. Simulation example and experimental results of speed DC servomechanism identification, by open loop and closed loop (direct method and two steps method), show the consistence and good accuracy of the estimate, important in *identification for control*.

Keywords— closed loop identification; parameter estimation; instrumental variable; adaptive algorithm; Householder orthogonal transformations.

1 Introduction

Often in the area of identification system, a common concern and important problem is that the input and output measurements may be contaminated by noise. Another source of random noise in the measured data is that the system to be identified is also driven by disturbance at some point. It is a common problem in closed loop identification where many of the identification methods, that work well in open loop, fail when applied directly to measured input-output data(Hjalmarsson, Gevers and Bruyne,1997)(Schrama and Hof,1995). The reason is the nonzero correlation between the input and the unmeasured output noise that is inevitable in adaptative control schemes. For low levels of noise by using least squares (LS) method, for example, may produce excellent estimates of the system parameters. However, with larger levels of noise may require some modifications in this method to overcoming the inconsistency problem induced by noise acting on the system. Many kinds of modified least square method have been developed such as the generalized least square (GLS) method, the extended least square (ELS) method and prediction error (PE) method, where the noise model needs to be estimated at same time as the system parameters are being estimated. Thus the results of these methods are inevitably dependent upon the accuracy of the noise model and some constraint conditions on it must be satisfied in these methods to obtain consistent parameter estimates(Broman and Anderson,1996)(Zhang ett all,1997). In general, however, it is very difficult to model the noise accurately and it is also hard to know a priori whether the noise model satisfies these conditions. To overcoming the bias problem without modelling the noise, the instrumental variables (IV) method can be developed. It provides a promising way to obtain consistent estimates which have certain optimal properties by choosing proper instrument variables (Wilson and Carnal,1993)(Broman and Anderson,1996).

This paper presents a weighted instrumental variable (WIV) algorithm based on orthogonal transformation via QR factorization to obtain the properties below presented. Simulation and experiments results show the efficiency of algorithm with the instrument variables proposed applied to direct and indirect identification of linear and nonlinear systems discussed in (Van den Hof and Schrama,1993) and its application to self-tuning control design, important in industrial control

2 Problem Formulation

Consider the ARX structure

$$y_t = b_1 u_{t-1} + \ldots + b_{nb} u_{t-nb} - a_1 y_{t-1} - \ldots \\ - a_{nc} y_{t-na} + \xi_t \qquad (1)$$

where u(t) and y(t) are the system input and output, respectively. ξ_t is an unknown noise disturbing the system. Denote :

$$\theta^T = (b_1, \ldots, b_{nb}, a_1, \ldots, a_{na}) \qquad (2)$$

$$\mathbf{a}_t^T = (u_{t-1}, \ldots, u_{t-nb}, -y_{t-1}, \ldots, -y_{t-na}) \qquad (3)$$

Then the system eqn. 1 can be expressed by following vectorial form :

$$\mathbf{Y}=\mathbf{A}\theta+\Xi \quad (4)$$

where

$$\mathbf{Y}^T=[\mathrm{y}_1,...,\mathrm{y}_n] \quad (5)$$

$$\mathbf{A}^T=[\mathbf{a}_1,...,\mathbf{a}_p] \quad (6)$$

$$\Xi^T=[\xi_1,...,\xi_n] \quad (7)$$

with p equal the dimension of the problem, i.e. $nb+na$ and n is the number of sample.

We seek to obtain a consistent parameter estimate for the parameter θ from the available observed data $\{y_t, u_t\}_1^n$ so that the error e_t between the measure output of the system y_t and the output of the associated model $\hat{y}_t$ is minimum in the least square sense, this is, the vector $\hat{\theta}$ that solves

$$\min\|\mathbf{A}\theta-\mathbf{y}\|_2^2 \quad (8)$$

or in *normal equations* form

$$\mathbf{A}^T\mathbf{A}\theta=\mathbf{A}^T\mathbf{Y} \quad (9)$$

3 Derivation of the algorithm

When studying the performance of numerical algorithms, the concepts of 'numerical stability' and 'conditioning' are of fundamental importance. The former is a property of the algorithm that is used for carrying out the computations, whereas the latter is a property associated with the computing problem and the data given for the problem. The numerical errors in any computation will depend on the stability of the algorithm used and the conditioning of the problem, and not the algorithm. The our algorithm uses orthogonal matrices to solve the least square problem in eqn.(9) by QR factorization. The use of orthogonal transformation for solving least squares problems is well established, as is the inadvisability of utilizing the normal equations (Bobrow and Murray,1993). The use of orthogonal transformation matrices is preferred because they are easy to invert, giving great accuracy and speed computationally, they are always perfectly conditioned and backward error analysis is simplified considerably when orthogonal transformations are used. The reason for this is that spectral and euclidean norms, which are the ones must commonly used in such analysis, are invariant under orthogonal transformations(Petel and Laub,1994).

The orthogonal Householder matrix is of the following form

$$\mathbf{H}=\mathbf{I}-2\frac{\mathbf{v}\mathbf{v}^T}{\|\mathbf{v}\|_2^2} \quad (10)$$

where $\mathbf{H}=\mathbf{H}^T$ and $\mathbf{H}=\mathbf{H}^{-1}$. Householder transformations are often used to annul block of elements in matrices or vectors by appropriately selecting the Householder vector $\mathbf{v}$ in eqn.(10). If $\mathbf{x}$ is a nonzero vector and $\mathbf{e_i}$ is an unit vector with 1 in the i-th position, then it can be shown that when

$$\mathbf{v}=\mathbf{x}\pm\|\mathbf{x}\|\mathbf{e_i} \quad (11)$$

then

$$\mathbf{H}\mathbf{x}=\mp\|\mathbf{x}\|\mathbf{e_i} \quad (12)$$

The vectors $\mathbf{v}$ and $\mathbf{x}$ are identical except for the i-th element. In our analysis, the explicit formation of the Householder matrix is not required, which is also in most cases.

3.1 Instrumental Variable Method

The solution of the IV method is that a $\mathbf{Z}$ matrix is defined so that it is uncorrelated with the noise and correlated with the input and output. Therefore the following conditions must be satisfied

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)\mathbf{Z}^T\Xi=0 \quad (13)$$

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)\mathbf{Z}^T\mathbf{X}=\mathbf{G} \quad (14)$$

where $\mathbf{G}$ is a nonsingular matrix. The eqn.(13)-(14) guarantee asymptotically unbiased parameter estimates.

In this paper, the set of instruments is chosen to be the delayed measurable inputs, i.e., the t-th row of the $\mathbf{Z}$ matrix is given by

$$\mathbf{z}_t=[\,u_{t-1}\quad u_{t-2}\quad \dots\quad u_{t-p}\,] \quad (15)$$

where p is equal the dimension of the problem, i.e. $nb+na$.

3.2 On-line Identification Algorithm (WIV)

In many applications, the structure of the model may be known, but its parameters may be known and changing with time because of change in operation conditions, aging of equipment, etc., rendering off-line parameter estimation techniques ineffective. Thus, this work was motivated by developing of an algorithm that provide frequent estimates of the parameters by properly processing the I/O data on-line and to adapt itself to possible variation of the parameters with time.

The interest problem may be couched as

$$\mathbf{Z}^T\mathbf{A}\theta=\mathbf{Z}^T\mathbf{Y} \tag{16}$$

where $\mathbf{Z}_{n\times p}$, $\mathbf{A}_{n\times p}$, $\theta_{p\times 1}$ and $\mathbf{Y}_{n\times 1}$ are the instrumental variable matrix, data matrix, parameters vector and output vector, respectively.

The eqn.(16) can be rewritten as

$$\mathbf{Z}^T\mathbf{W}\mathbf{A}\theta=\mathbf{Z}^T\mathbf{W}\mathbf{Y} \tag{17}$$

where $\mathbf{W}_{n\times n}$ and $\mathbf{W}_n=diag\left(\lambda^{n-1},\lambda^{n-2},...,1\right)$, with $0<\lambda<1$. The scalar λ is known as the *forgetting factor* and it is used to place less weight on past data.

Developing both sides in eqn.(17), as **Z**, **W**, **A** and **Y** are known, result

$$\mathbf{S}\theta=\mathbf{b} \tag{18}$$

where $\mathbf{S}_{p\times p}=\mathbf{Z}^T\mathbf{W}\mathbf{A}$ and $\mathbf{b}_{p\times 1}=\mathbf{Z}^T\mathbf{W}\mathbf{Y}$. It is worth emphasizing that the resulting order of the **S** matrix and of the **b** vector are lower than order of the **A** matrix and of the **Y** vector, because *p* is equal to the number of parameters that will be estimated, implying less computational effort and, consequently, greater speed to solution of θ.

Generically, the **Z**, **W**, **A** matrices and the **Y** vector are given by

$$\mathbf{Z}^T=\begin{bmatrix} u_0 & u_1 & \cdots & u_{n-1} \\ u_{-1} & u_0 & \cdots & u_{n-2} \\ \bullet & \bullet & \cdots & \bullet \\ u_{-p+1} & u_{-p+2} & \cdots & u_{n-p} \end{bmatrix};$$

$$\mathbf{W}=\begin{bmatrix} \lambda^{n-1} & 0 & . & . & 0 \\ 0 & \lambda^{n-2} & 0 & . & . \\ . & 0 & . & 0 & . \\ . & . & 0 & \lambda & 0 \\ 0 & . & . & 0 & 1 \end{bmatrix};$$

$$\mathbf{A}=\begin{bmatrix} u_0 & u_{-1} & \cdots & u_{1-nb} & -y_0 & -y_{-1} & \cdots & -y_{1-na} \\ u_1 & u_0 & \cdots & u_{2-nb} & -y_1 & -y_0 & \cdots & -y_{2-na} \\ \bullet & \bullet & \cdots & \bullet & \bullet & \bullet & \cdots & \bullet \\ u_{n-1} & u_{n-2} & \cdots & u_{n-nb} & -y_{n-1} & -y_{n-2} & \cdots & -y_{n-na} \end{bmatrix}$$

and

$$\mathbf{Y}=\begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ y_n \end{bmatrix}$$

Hence, $\mathbf{S}_{p\times p}=\mathbf{Z}^T\mathbf{W}\mathbf{A}$ result

$$\mathbf{S}=\begin{bmatrix} \sum_{t=1}^{n} u_{t-1}^2\lambda^{n-t} & \cdots & \sum_{t=1}^{n} u_{t-1}u_{t-nb}\lambda^{n-t} & -\sum_{t=1}^{n} u_{t-1}y_{t-1}\lambda^{n-t} & \cdots & -\sum_{t=1}^{n} u_{t-1}y_{t-na}\lambda^{n-t} \\ \sum_{t=1}^{n} u_{t-2}u_{t-1}\lambda^{n-t} & \cdots & \sum_{t=1}^{n} u_{t-2}u_{t-nb}\lambda^{n-t} & -\sum_{t=1}^{n} u_{t-2}y_{t-1}\lambda^{n-t} & \cdots & -\sum_{t=1}^{n} u_{t-2}y_{t-na}\lambda^{n-t} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \sum_{t=1}^{n} u_{t-p}u_{t-1}\lambda^{n-t} & \cdots & \sum_{t=1}^{n} u_{t-p}u_{t-nb}\lambda^{n-t} & -\sum_{t=1}^{n} u_{t-p}y_{t-1}\lambda^{n-t} & \cdots & -\sum_{t=1}^{n} u_{t-p}y_{t-na}\lambda^{n-t} \end{bmatrix} \tag{19}$$

and $\mathbf{b}_{p\times 1}=\mathbf{Z}^T\mathbf{W}\mathbf{Y}$ is

$$\mathbf{b}=\begin{bmatrix} \sum_{t=1}^{n} u_{t-1}y_t\lambda^{n-t} \\ \sum_{t=1}^{n} u_{t-2}y_t\lambda^{n-t} \\ \bullet \\ \sum_{t=1}^{n} u_{t-p}y_t\lambda^{n-t} \end{bmatrix} \tag{20}$$

From eqn.(19)-(20), we can observe that the elements of the **S** matrix and of the **b** vector are summations that depend of the actual and immediately former values, based on the dimension of the problem, of the input and output measures. This imply in generating, directly, i.e., in each sample, **S** and **b**, without need of a priori batch matricial operations, as in eqn.(17), with advantage that the order problem is lower to application of the QR factorization.

Thus, the problem may be couched as that of finding the solution of

$$\underset{\hat{\theta}}{\text{minimize}} \| \mathbf{S}\theta - \mathbf{b} \|_2^2 \quad (21)$$

Applying QR factorization via House-holder orthogonal transformations, we have

$$\underset{\hat{\theta}}{minimize} \| \mathbf{Q}^T\mathbf{S}\theta - \mathbf{Q}^T\mathbf{b} \|_2^2 \quad (22)$$

and

$$\underset{\hat{\theta}}{minimize} \| \mathbf{R}\theta - \mathbf{d} \|_2^2 \quad (23)$$

where $\mathbf{Q}_{p\times p}$ is an orthogonal matrix, $\mathbf{R}_{p\times p}$ is an upper triangular matrix and $\mathbf{d}_{p\times 1}$ is a resulting vector. Hence, the minimizer of eqn.(21) may be found by solving $\mathbf{R}\hat{\theta} = \mathbf{d}$ by back substitution. The algorithm receive an initial batch data to initial estimation and the updating is obtained for simple acquisition of input and output data and insert it into summations of the matrix **S** and of the vector **b**, this is, in the k-th sample, we have

$$\mathbf{S}_{new} = \mathbf{S} + \lambda \begin{bmatrix} u_k^2 & \cdots & u_k u_{k-} & -u_k y_k & \cdots & -u_k y_{k-na+1} \\ u_{k-1}u_k & \cdots & u_{k-1}u_k & -u_{k-1}y_k & \cdots & -u_{k-1}y_{k-na+1} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ u_{k-p+1}u_k & \cdots & u_{k-p+1}u & -u_{k-p+1}y_k & \cdots & -u_{k-p+1}y_{k-na+1} \end{bmatrix} \quad (24)$$

and

$$\mathbf{b}_{new} = \mathbf{b} + \lambda \begin{bmatrix} u_k y_k \\ u_{k-1} y_k \\ \bullet \\ u_{k-p+1} y_k \end{bmatrix} \quad (25)$$

The following is the algorithm :

Step 1 : Define a number of input and output measures to automatic initial estimation.

Step 2 : Generating the **S** matrix and **b** vector from eqn.(19)-(20).

Step 3 : Applying QR factorization by House-holder orthogonal transformations to generate eqn.(23).

Step 4 : Solving eqn.(23) by back substitution.

Step 5 : Obtaining a new input and output measure.

Step 6 : Generating the new **S** matrix and the new **b** vector from eqn.(24)-(25).

Step 7 : Go to step 3

4 Results

In this section, we present a simulation and experimental results in order to illustrate the advantages of the proposed method in closed loop identification aplications as a basis to identification for control.

4.1 Simulation Results

In order to illustrate the consistency and accuracy of the algorithm, consider the same system discussed in (Van den Hof and Schrama,1993). In our application, we added a nonlinear operator as shown the Fig. 1. The noise signal *v* and the reference signal *r* are independent unit variance zero mean random signals. The controller is designed in such a way that the closed loop transfer function has a denominator polynomial $(z-0.3)^2$.

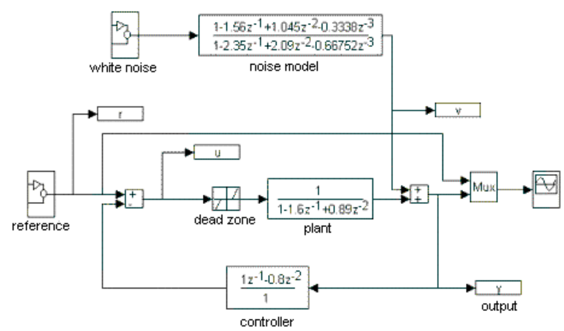

Figure 1. Closed loop identification problem.

To implement the proposed algorithm, λ is taken as 0.95, the 25 input and output measures were utilized to initial estimation and the total of points was 2048.

The direct and the two-step identification strategy is applied with and without the nonlinear operator. The results are the following:

- **Direct method**

The Fig. 2 shows the Bode plot of the plant transfer function (blue), the estimate obtained with nonlinearity (green) and the estimate obtained without nonlinearity (red).

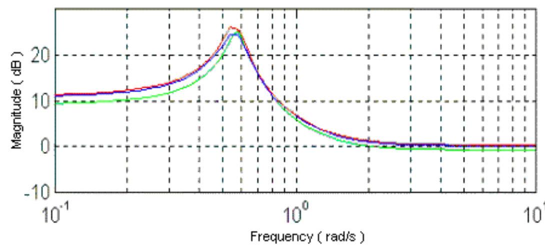

Figure 2. Closed loop identification by direct method.

The results clearly show the good accuracy of the estimated models and consistency of the proposed algorithm which is important to identification for control context.

- **Indirect method (two – steps)**

In the first step, the sensitivity function is estimated. In the second step, the transfer function of the plant is obtained based on the sensitivity function [10]. The Fig. 3 show the Bode amplitude plot of the exact sensitivity function (blue) and the estimated sensitivity function with nonlinearity (green) and the estimated sensitivity function without nonlinearity (red). The Fig. 4 shows the Bode amplitude plot of the plant transfer function (blue), the estimated model with nonlinearity (green) and the estimated model without nonlinearity (red).

The results show the accuracy of the proposed algorithm applied to direct and indirect closed loop identification of linear and non-linear systems with the advantage that it is don't need to model of the noise.

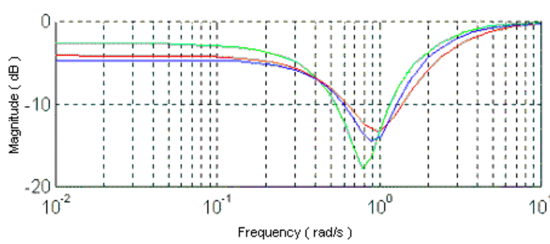

Figure 3 Sensitivity function estimate.

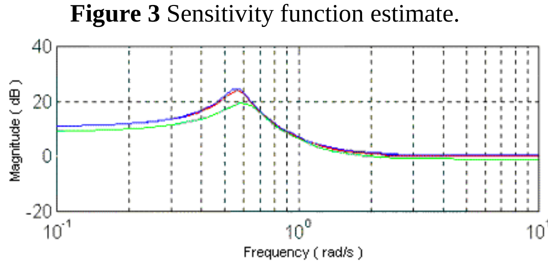

Figure 4. Closed loop identification by two-step strategy.

4.2 Experimental Results

In system identification, emphasis has long been on aspects of consistency and efficiency, related towards the reconstruction of the "real plant" that underlies the measurement data. However, in real situations, models that are identified from data will generally be contaminated with errors due to both aspects of bias (undermodelling) and variance (Hjalmarsson, Gevers and Bruyne,1997)(Schrama and Hof,1995). The our experiment is to identify speed DC servomechanism of our control and automation laboratory. This identification process is divided in two steps :

Step 1 : Open loop identification as shown on Fig. 5.
Step 2 : Closed loop identification utilizing the direct method, this is, we ignore the feedback and identify the open loop system using measurements of the input and the output as shown on Fig. 6.

In the 2 step we use the proportional controller of $K_p = 1$, to illustrate the application of the algorithm to closed loop identification and we are concerned only with the identificated model analysis.

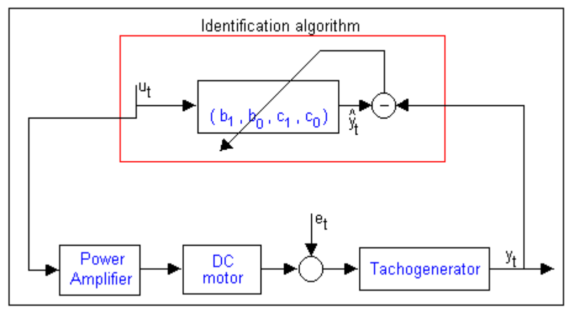

Figure 5. Step 1 : Open loop identification

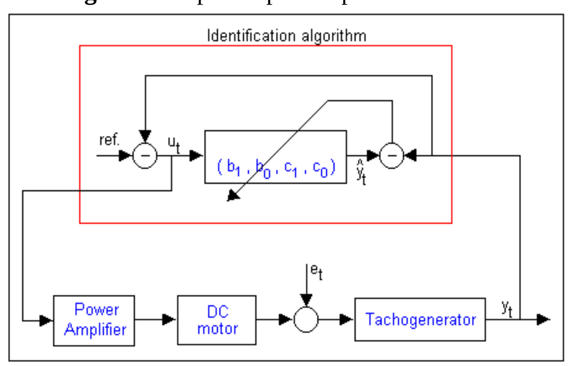

Figure 6. Closed loop identification by direct method.

To this experiment λ is taken as 0.95, 25 pairs of input-output data were utilized to initial estimation, the sample period T is taken 10ms and the total of points is 600. The input and reference signals is taken as the voltage of 4.0V. The Tab. 2 shows the obtained comparative results of the estimates of parameters in open loop and by closed loop identification methods presented.

Table 1. Parameters using the developed algorithm to open loop and direct closed loop identification.

Parameters	Open loop	Direct method
a_1	-0.227422	-0.165243
a_0	-0.570980	-0.579160
b_1	0.082230	0.094971
b_0	-0.017469	-0.010481

Considering the open loop results as true parameters, we can note that the accuracy of the direct method. It is important emphasizing the effect of the nonlinearity from power amplifier circuit and the noise signals from the acquisition data system.

The Fig. 7 shows the estimated and real curves of the two cases.

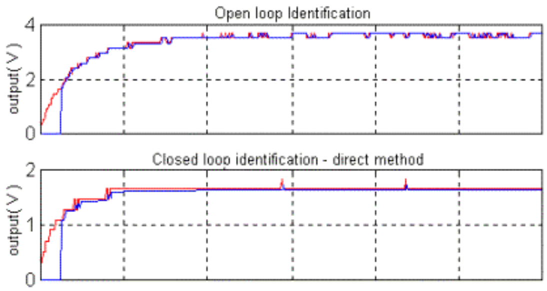

Figure 6. Output estimation to the two cases above mentioned with the real curve in red and the estimated curve in blue.

The results clearly show the accuracy of the estimation by direct method, this is, the real and estimated curves are practically equals and, thus, it shows the applicability of the algorithm to closed loop identification in the sense that the identified model represent the dynamic behavior of the plant in closed loop which is important in identification for control.

5 Conclusion

An adaptative instrumental variable algorithm has been developed for system identification in this paper. A choice of instruments was presented and the QR factorization by Householder orthogonal transformation was implemented. The consistency of the developed Weighted Instrumental Variable (WIV) method has been established and simulation results show an accurate unbiased parameter estimation. The simulation results show the advantage of the algorithm to closed loop identification of linear and nonlinear systems and direct and indirect (two steps) methods showed great performances to identify the plant model based on closed loop data. The algorithm was applied to open loop and direct closed loop identification of the speed DC servomechanism which belongs to our control and automation laboratory to extend its applicability to control schemes where the controller design is done iteratively based on identified model, this is, it can be also applied to self-tuning control design, which is important in industrial control.

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