Qualitative Reasoning

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Preface

This volume contains the papers presented at the 29th International Workshop on Qualitative Reasoning held on July 11, 2016 in New York.

The Qualitative Reasoning (QR) community develops qualitative representations to understand the world from incomplete, imprecise, or uncertain data. Our qualitative models span natural systems (e.g., physics, biology, ecology, geology), social systems (e.g., economics, cultural decision-making), cognitive systems (e.g., conceptual learning, spatial reasoning, intelligent tutors, robotics), and more.

The QR community includes researchers in Artificial Intelligence, Engineering, Cognitive Science, Applied Mathematics, and Natural Sciences, commonly seeking to understand, develop, and exploit the ability to reason qualitatively. This broadly includes:

- Developing new formalisms and algorithms for qualitative reasoning.

- Building and evaluating predictive, prescriptive, diagnostic, or explanatory qualitative models in novel domains.

- Characterizing how humans learn and reason qualitatively about the (physical) world with incomplete knowledge.

- Developing novel, formal representations to describe central aspects of our world: time, space, change, uncertainty, causality, and continuity.

The International Workshop on QR provides a forum for researchers from multiple perspectives to share research progress toward these goals. Topics of interest include:

- Qualitative modeling in physical, biological and social sciences, and in engineering.

- Representations and techniques for QR.

- Methods that integrate QR with other forms of knowledge representation, including quantitative methods, machine learning and other formalisms.

- Using QR for diagnosis, design, and monitoring of physical systems.

- Applications of QR, including education, science, and engineering.

- Cognitive models of QR, including the use of existing QR formalisms for cognitive modeling and results from other areas of cognitive science for qualitative reasoning.

- Using QR in understanding language, decision-making, sketches, images, and other kinds of signals and data sources.

- Formalization, axiomatization, and mathematical foundations of QR.

July 2, 2016   Bert Bredeweg
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Collaborative Communication of Qualitative Spatial Perceptions for Multi-Robot Systems

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Abstract

The goal of this work is to develop a collaborative communication system of spatial perceptions for vision-based multi-robot systems using qualitative spatial reasoning, where the representation of the domain is built upon the perspective of the Elevated Oriented Point Algebra ($\mathcal{EOPRA}$) and the reasoning itself is made by a combination between the Oriented Point Algebra ($\mathcal{OPRA}$) and a quantitative triangulation. The motivation of using qualitative information is to obtain a level of abstraction closer to the human categorisation of space and, also, to have a more effective way of interaction between robots and humans. Results allowed us to conclude that the method proposed is an effective way to address the high-level communication between only-robots agents or between robots and humans by using some spatial prepositions.

Introduction

Robots will soon achieve a level of electrical and mechanical development that would allow their insertion into the common (non-industrial) human environment. This fact brings atop the importance of developing robots that can interact with humans in a seamless way (Dylla, Kreutzmann, and Wolter 2014). To this end, the present paper describes our investigation on the development of a collaborative communication system of spatial perceptions for vision-based multi-robot systems using qualitative spatial information.

The use of qualitative representations is motivated by the fact that humans do not normally use numerical descriptions to talk about the commonsense space, so a seamless human-robot interaction implies a non-metrical representation of their common environment. Besides, there are cases where communicating qualitative relations are more effective than metrical information. For instance (Freksa 1991), imagine an aquarium full of fishes and two observers, one observer wants to point a particular fish to the other. Let's assume that there is only one red stone inside the aquarium. Pointing to this particular fish in terms of metric information (e.g. "the fish is 10 cm away from the aquarium's left wall, 5 cm from its bottom, 8 cm from the rear wall and 1 m away from you") is much harder to understand than pointing to it in a purely qualitative way (e.g. "the fish

near the red stone"). In order to deal with qualitative representations, this paper assumes formalisms developed in the area of Qualitative Spatial Reasoning (QSR) (Ligozat 2013; Cohn and Renz 2007). QSR is a subfield of Knowledge Representation in Artificial Intelligence that develops formalisations of space by means of qualitative relations. The use of qualitative methods allow reasoning with incomplete knowledge (Renz and Nebel 2007) and facilitates meaningful abstractions of the physical world (Moratz 2006). From qualitative representations of space, high-level communication is favoured. This promotes the application of QSR to Multi-Robot Systems in human environments.

This work assumes the interaction of groups of robots from the RoboCup Soccer Humanoid League as a domain where the ideas developed here are evaluated. In the present paper the robots collaborate by sharing their individual visual observations of a scene with each other in order to enhance their knowledge about the environment their are immersed in. Two experiments were conducted: in the first, the group of robots had to answer spatial queries using the information perceived by each robot. This information was shared among the group members and inference over qualitative relations was used to combine the multiple pieces of data. In the second experiment, the qualitative calculus was used to communicate the observations of one robot about a target that was occluded with respect to another robot.

The collaborative communication system proposed in this paper uses the discretisation of the Elevated Oriented Point Algebra with granularity 6 ($\mathcal{EOPRA}_6$) (Moratz and Wallgrün 2012). The $\mathcal{EOPRA}_m$ notation is derived from $\mathcal{OPRA}_m$ (Moratz 2006) and allows a joint representation of qualitative direction and distance between points. The reasoning of this paper is a combination of $\mathcal{OPRA}_m$ and a quantitative triangulation.

Qualitative Spatial Reasoning

One of the main challenges of QSR is the development of formal systems to represent the spatial configuration of entities in purely qualitative terms, also permitting reasoning using this representation (Cohn and Renz 2007; Dylla 2009). These formal systems use a limited amount of qualitative categories to represent the possible spatial relations between entities (Renz and Nebel 2007). Applications of QSR vary from high-level computer vision, seman-

tic of spatial propositions, reasoning about commonsense knowledge, geographical information systems, among others (Cohn and Renz 2007). In particular, the formalism named Oriented Point Algebra ($\mathcal{OPRA}_m$) (Moratz 2006; Mossakowski and Moratz 2012) has been very influential for representing and reasoning about objects with intrinsic fronts (Dylla et al. 2007), such as cars and boats (Dylla 2009), but also robots (Moratz 2006; Dylla, Kreutzmann, and Wolter 2014). This formalism is essential in the development of the present work and it is described as follows.

Oriented Point Algebra ($\mathcal{OPRA}_m$)

The Oriented Point Algebra with granularity m ($\mathcal{OPRA}_m$) is a qualitative calculus in which objects are represented as oriented points, that are represented by Cartesian coordinates, x and y, and an orientation, θ. Each point defines a relative reference frame of granularity m where $m \in \mathbb{N}$. This granularity is used in order to obtain the angular resolution, which is equal to $\frac{2\pi}{2m}$ (Mossakowski and Moratz 2012).

In $\mathcal{OPRA}_m$, if the Cartesian coordinates of two oriented points, A and B are different (cf. Figure 1), the relationship between the points is represented by $A_m \angle_i^j B$, which means: given the granularity m, the relative position of B with respect to A is described by i and the relative position of A with respect to B is j. For example, the relation between A and B, in Figure 1, is $A_4 \angle_{11}^1 B$, meaning that A is in the sector 1 of B; B is in the sector 11 of A, and 4 is the granularity of the relative frame. Such as in other methods of QSR, the $\mathcal{OPRA}_m$ reasoning is done through a composition table, where this table is constructed with the set of all relations between three-oriented points, for example, $A_m \angle_i^j B$, $B_m \angle_k^l C$ and $A_m \angle_s^t C$, where i, j, k, l, s, t are variables that describe the relations between the points (Moratz 2006).

For example, Figure 1 shows the composition of the relations $A_4 \angle_{11}^1 B$ and $B_4 \angle_{13}^9 C$ from which the relation between the points A and C can be inferred.

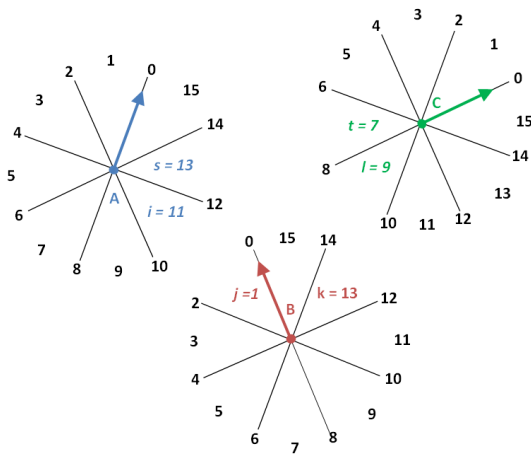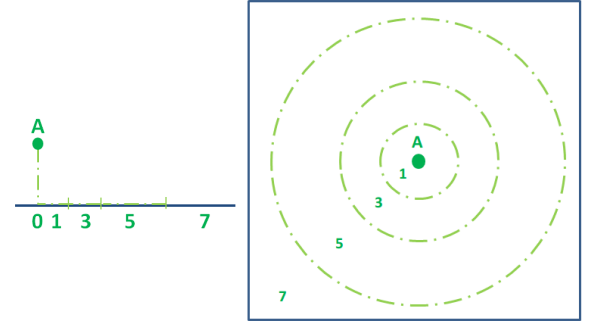$\mathcal{OPRA}_m$ works only with orientation, however, in the

(a) Side view.   (b) Bird's eye view.

Figure 2: Qualitative distances with $m = 4$: $\delta \times 0$, $\delta \times 1/2$, $\delta \times 1$ and $\delta \times 2$ (Moratz and Wallgrün 2012).

real world, another important spatial information is distance. Distance can be defined qualitatively by using the idea of elevated point, described below.

Elevated Point as Reference for Qualitative Distance

A definition of relative distances, based on local distance references (*elevations*), was proposed by (Moratz and Wallgrün 2012). Elevations are defined by the height of observers, whose projection in the 2D plane defines a circle around the observer's locations, that is used as a distance reference (Gibson 1986). The size of this projection is represented by δ, and all the distance ratios are calculated taking into consideration m and δ (Dorr and Moratz 2014). Granularity (also represented by m in the distance representation) also applies to elevations in order to provide the appropriate level of abstraction for distance relations. Distance relations between two points A and B are represented as $A \, {}_m\bigcirc_e^f B$, where e represents the relative distance of B with respect to A and f, the relative distance of A with respect to B.

The function $b_A(e)$, shown in the Equation 1, calculates the boundaries of qualitative distances around the elevated point A, where $0 \leqslant e \leqslant 2m$ and e must be an even number (Moratz and Wallgrün 2012). Figure 2 shows an example of a qualitative distance for $m = 4$.

$$b_A(e) = \begin{cases} \infty & \text{if } e = 2m, \\ \frac{e\delta_A}{m} & \text{if } e \leqslant m, \\ \frac{m\delta_A}{2m-e} & \text{otherwise.} \end{cases} \tag{1}$$

Elevated Oriented Point Algebra ($\mathcal{EOPRA}_m$)

$\mathcal{EOPRA}_m$ is an extension of $\mathcal{OPRA}_m$ that includes qualitative distances as elevated points. The $\mathcal{EOPRA}_m$ notation is derived from $\mathcal{OPRA}_m$, allowing a joint representation of qualitative direction and distance between two points as: $A_m \angle_i^{j} \, {}_e^f B$, where m is the common arbitrary granularity between distance and direction, i and j are orientation relations, and e and f are distance relations. Figure 3 represents the relation $A_4 \angle_{11}^1 {}_3^5 B$ in $\mathcal{EOPRA}_m$ for two points A and B with distinct elevations.
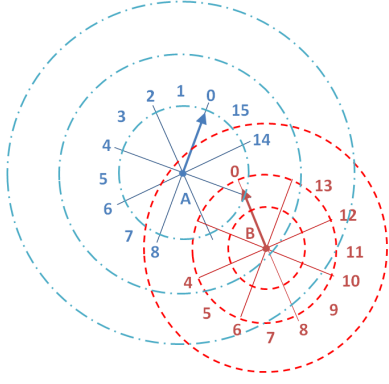

Figure 1: Composition of $A_4 \angle_{11}^1 B$ with $B_4 \angle_{13}^9 C$ can result in $A_4 \angle_{13}^7 C$, for example.

Figure 3: $\mathcal{EOPRA}_4$ relation $A \; _4\angle^1_{11} \; ^5_3 \; B$.

# Collaborative Communication of Spatial Perceptions for Multi-Robot Systems

This section describes our proposal of a collaborative communication system of spatial perceptions for vision-based multi-robot systems, where the representation of the domain is built upon the perspective of the Elevated Oriented Point Algebra ($\mathcal{EOPRA}$) and the reasoning itself is made by a combination between the Oriented Point Algebra ($\mathcal{OPRA}$) and a quantitative triangulation. $\mathcal{EOPRA}_m$ discretisation is suitable for this purpose since it treats both direction and distance and allows for relative spatial perception communication, whereby a robot can locate itself "*through the eyes*" of the other robots in the domain.

In this work the granularity $m = 6$ was chosen. $\mathcal{EOPRA}_6$ discretisation is exemplified in Figure 4a. Also, the numerical regions defined were labelled by means of spatial prepositions, as shown in Figure 4b, where fr, $l\text{-}fr$, l, $l\text{-}b$, b, $r\text{-}b$, r and $r\text{-}fr$ stand for $front$, $left\text{-}front$, $left$, $left\text{-}back$, $back$, $right\text{-}back$, $right$ and $right\text{-}front$, respectively. Likewise, at, vc, c, f, vf and ft stand for at, $very\text{-}close$, $close$, far, $very\text{-}far$ and $farthest$.

The multi-robot collaboration method proposed allows a robot to answer spatial queries even if the robot is not directly involved in the relation queried, or if it has incomplete knowledge of the domain. In this work, inference processes of directions and distance are made separately.

Due to the poverty conjecture (Forbus, Nielsen, and Faltings 1991), it is known that is, in fact, impossible to achieve a purely qualitative spatial reasoning mechanism (Cohn and Renz 2007). Thereby, distance inference is accomplished by quantitative triangulation using the law of cosines. This is possible because distances are quantitatively estimated before being discretised by means of elevations. In the same way, quantitative data are going to be used for restricting the number of possible relations during the direction inference.

Direction inference is based on $\mathcal{OPRA}_m$. However, $\mathcal{OPRA}_m$ algorithm (Mossakowski and Moratz 2012) only checks whether a composition made by the relations of the oriented points holds, i.e., it does not directly infer a composition. So, we introduce Algorithm 1, which allows a systematic way for inferring the set of possible orientations s, or s and t, in $\mathcal{OPRA}_m$ composition of relations $A_m\angle^j_i B$,
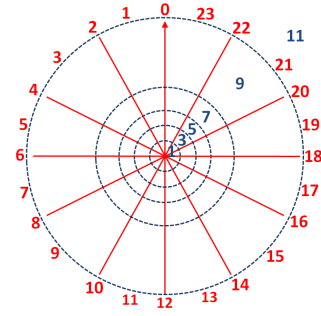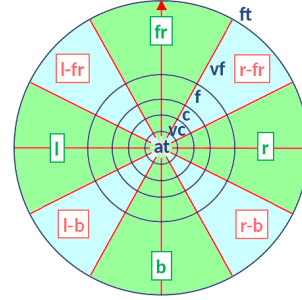
(a) $\mathcal{EOPRA}_6$

(b) Translating the numerical regions to spatial prepositions

Figure 4: $\mathcal{EOPRA}_6$: proposed discretisation.

$B_m\angle^l_k C$ and $A_m\angle^t_s C$. This algorithm checks which values s can assume, when t is given; or which values of s and t, when t is not given, and returns all compositions that hold.

Algorithm 1 may return a disjunction of relations as a result of a given composition. It is possible to reduce the number of possible relations in this disjunction by using triangulation (as represented in Algorithm 2). An example of obtaining this restriction is shown in Figure 5, where the blue robot should locate the green robot with respect to the red one. By using $\mathcal{OPRA}_m$ inference method, and assuming that t is not given, s could assume any of the following values: $0, 1, 2, 3, 17, 18, 19, 20, 21, 22, 23$, which means, fr, $l\text{-}fr$, r and $r\text{-}fr$. The blue robot can easily obtain the angle β, so it can calculate the angle α using the law of cosines. In the example shown in Figure 5, the quantitative angle α is equal to 46°, however, as β is negative, α should also be negative in order to form a triangle. After obtaining α, this angle is discretised according to $\mathcal{OPRA}_6$ definitions, resulting in the relation $i_\alpha = -3$. Then, this new relation is added up to the relation i, where $i = 3$, leading to $i_{restr} = 0$. Now Algorithm 2 checks if i_{restr} is an even number. If so, this region is transformed into a set comprised of two odd regions ($[i_{restr} + 1, i_{restr} - 1]$). If not, i_{restr} is kept. If this odd region, or the items of the set of two regions, is contained in the set of relations inferred by $\mathcal{OPRA}_6$, then it becomes the output of the system; if not contained, the system returns failure (i.e. a contradiction has been found).

The next Section will show some preliminary experiments with our proposed $\mathcal{OPRA}_6$ combined with a quantitative triangulation that uses the $\mathcal{EOPRA}_6$ representation for performing collaborative communication of spatial perceptions for multi-robot systems.

Algorithm 1 Inferring the set of relations $\hat{s}$ or $\hat{s}$ and $\hat{t}$ of $\mathcal{OPRA}_m$ for non-coincident points.

Function $OpraInference(m, i, j, k, l, t)$

1: **if** $t = \emptyset$ **then**
2:    **for** $s_{test} = 0$ to $4m$ **do**
3:       **for** $t_{test} = 0$ to $4m$ **do**
4:          **if** $opra(m, i, j, k, l, s_{test}, t_{test})$ **then**
5:             add s_{test} to $\hat{s}$ and add t_{test} to $\hat{t}$
6:          **end if**
7:       **end for**
8:    **end for**
9:    **return** $\hat{s}, \hat{t}$
10: **else**
11:    **for** $s_{test} = 0$ to $4m$ **do**
12:       **if** $opra(m, i, j, k, l, s_{test}, t)$ **then**
13:          add s_{test} to $\hat{s}$
14:       **end if**
15:    **end for**
16:    **return** $\hat{s}$
17: **end if**

Function $opra(m, i, j, k, l, s, t)$

18: **if** $\exists\ 0 \leqslant u, v, w < 4m.\ turn_m(u, i, -s)\ \wedge\ turn_m(v, k, -j) \wedge turn_m(w, t, -l) \wedge triangle(u, v, w)$ **then**
19:    **return** $True$
20: **end if**

Function $turn_m(o, p, q)$

21: **if** $|(o + p + k + 2m) \bmod 4m) - 2m| \leqslant (o \bmod 2) \times (p \bmod 2)$ **then**
22:    **return** $True$
23: **end if**

Function $triangle_m(u, v, w)$

24: **if** $turn_m(u, v, w - 2m) \wedge (u, v, w) \neq (2m, 2m, 2m) \wedge sign_m(u) = sign_m(v) = sign_m(w)$ **then**
25:    **return** $True$
26: **end if**

Function $sign_m(q)$

27: **if** $(q \bmod 4m = 0) \vee (q \bmod 4m = 2m)$ **then**
28:    **return** 0
29: **else if** $(q \bmod 4m < 2m)$ **then**
30:    **return** 1
31: **else**
32:    **return** -1
33: **end if**

Experiments and Results

Experiments were made in two phases: first, the tests were performed in a simulated environment, using the RoboFEI-HT Soccer Simulator, in order to evaluate the proposed method with a considerably quantity of data points. Then, the method was validated in real humanoid robots.

Each phase was comprised of two experiments, used for evaluating the method proposed in this paper. The first experiment involves three robots, where each robot has to answer queries about the location of the other robots with respect to itself, or with respect to the other agents. In the second experiment, two robots have to locate a ball in a soccer
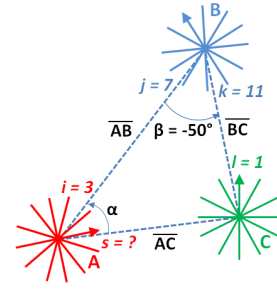

Figure 5: $\mathcal{OPRA}_6$ restricted by quantitative triangulation.

field, according to their own positions. However, the target is only perceived by one robot, but not by the other (i.e., the ball may be occluded, out of the field of view, or the robot might have a faulty sensor). The inference method proposed in this work is used in this case in order to allow the latter robot to locate the ball, using the observation provided by the former and the relative locations of both robots with respect to each other.

In all experiments conducted the robots were dressed with distinct colours, so that colour segmentation could be used to identify each agent. Orientation was obtained from the position of the motor in charge of the pan movement in robot's head. Distance was estimated by approximation functions, since all the sizes of the robots and other domain objects are known. The communication between the robots was conducted via broadcast using the User Datagram Protocol (UDP). The elevation δ, used for discretising the distances, was set to 1 meter, that is approximately twice the robot's

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Algorithm 2 Restricting the set of $\hat{s}$ relations by the quantitative triangulation.

Function $RestrictingOpra(m, \hat{s}, \alpha)$

1: $i_{aux} = DiscretizeToOpra(\alpha)$
2: $i_{restr} = (i + i_{aux}) \bmod (4m)$
3: **if** i_{restr} = even number **then**
4:    $\hat{c} = [i_{restr} + 1, i_{restr} - 1]$
5: **else**
6:    $\hat{c} = [i_{restr}]$
7: **end if**
8: **for** $n = 0$ to $len(\hat{s})$ **do**
9:    **for** $x = 0$ to $len(\hat{c})$ **do**
10:       **if** $s_n = c_x$ **then**
11:          add s_n to $\hat{a}$
12:       **end if**
13:    **end for**
14: **end for**
15: **if** $\hat{a} = empty$ **then**
16:    **return** $fail$
17: **else**
18:    **return** $\hat{a}$
19: **end if**

Function $DiscretizeToOpra(\alpha)$

20: $i_\alpha = round(angle/(180/m) * 2)$
21: **return** i_α

Figure 6: RoboFEI-HT Soccer Simulator.

(a) Without visible $\mathcal{EOPRA}_6$ discretisation (b) With visible $\mathcal{EOPRA}_6$ discretisation

Figure 7: RoboFEI-HT Soccer Simulator: one of the robots' position during the first experiment.
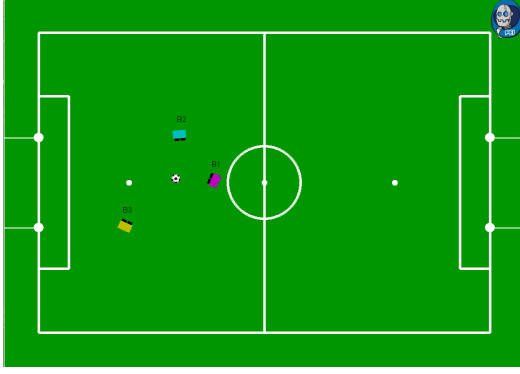
height. This value provides the appropriate distance reference for the chosen domain.

Simulated Environment

The soccer simulator, shown in Figure 6, used for performing the first phase of the experiments was designed and developed by the authors, in order to simulate the control and the vision system of the real humanoid robots. One of the qualities of this simulator is that code made for it can be also used in our real robots.

The simulator simulated Gaussian errors in the vision system for both, directions and distances, with standard deviation of 2° and 10 cm respectively.

Experiment 1: Communication Effectiveness with Three Robots. This first experiment analyses the effectiveness of the inference process and check the behaviour of the overall system, including the vision system. The evaluation was conducted by verifying spatial queries such as "$l(x, red\_robot)$" or "$f(y, red\_robot)$", respectively, "which is the robot x that is on the left of the red_robot?" and "which is the robot y that is far from the red_robot?".

The queries were broadcast by a human agent to all robots in the experiment via UDP. Every robot had to answer every query, even if the agent is not a variable in a query. The inference process is then performed by the robots to allow the attainment of the relative relations of direction and distance.

Robots were randomly arranged 30 times for each question, and the queries were always made in relation to the red robot (R). The inferences were made by the blue robot (B) and by the yellow robot (Y). An inference consisted in finding the relation of distance and direction that holds. Figure 7 depicts the simulator with three robots positioned according to the $\mathcal{EOPRA}_6$ relation $R_6\angle_3^{21}Y$, $Y_6\angle_3^{23}B$, and $R_6\angle_{23}^{1}B$.

For example, considering only direction, the blue robot (B) can infer its own location with respect to the red robot (R) by using the following information: (i) $R_6\angle_i^jY$, (ii) $Y_6\angle_k^lB$, and (iii) $R_6\angle_s^tB$, where (iii) is obtained by the composition of (i) and (ii). In these relations i is reported by R; j and k are reported by the yellow robot (Y); l and t are obtained from B's sensor data; and s is unknown. Considering the arrangement shown in Figure 7, we can say that the blue robot has received $i = 3$, $j = 21$, and $k = 3$, whereas it has perceived $l = 23$ and $t = 1$. It can then infer the
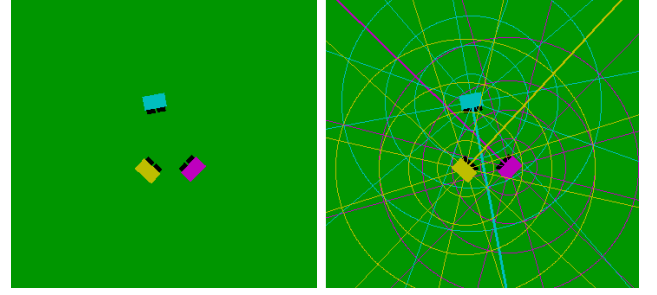
relation $s = 23$ using the $\mathcal{OPRA}_m$ inference restricted by triangulation. This represents the location of B w.r.t. R.

From the set of answers obtained, precision, recall and accuracy were calculated. Table 1 shows the rates obtained for direction-only queries; Table 2 shows the values for distance-only queries; and, Table 3 shows the rates for combined queries. Even considering the noise purposely added in the vision system, and the inaccuracies found during $\mathcal{OPRA}_m$ inference, the results show a precision of above 80% in most cases, as well as the recall.

The lower precision results were found for the queries that involves the small relations of direction, i.e., r-b and l-fr. This happens because the frontiers of this relations are closer to each other, so it is easier for the inference process to return a wrong region.

Experiment 2: Communication of Spatial Perceptions to Handle with Occlusion. In the second experiment, the blue robot (B) uses the proposed reasoning for communicating the relative location of a ball (O) to the red robot (R), that cannot see the target due to occlusion (Figure 8). This information is then communicated to R.

The blue robot was able to see both the ball and the red robot, whereas the red robot could only see the blue robot. The set of relations obtained by the blue robot is: $\{R_6\angle_i^jB, B_6\angle_k^lO, R_6\angle_s^tO\}$. During this experiment both s and t are inferred, even if only s is necessary. As the ball does not have an intrinsic orientation, we assumed that it is oriented toward the blue robot, i.e. $l = 0$.

This experiment was conducted by randomly positioning the ball 30 times in different positions inside of each qualitative region of the robot R. Then, the blue robot (B) inferred the ball position, i.e. direction and distance, in relation to the red one (R), using the qualitative inference system restricted by triangulation. As the discretisation of direction is symmetric, only the regions fr, l-fr, l and l-b were chosen for being evaluated. So, the results of the ball's position inference, made by the blue robot (B) are presented in two confusion matrix: Table 4 for direction and Table 5 for distance. Each column of the tables represents the inference made while the rows represent the actual position of the ball w.r.t. the red robot (R). It is possible to notice that, as well as seen in the last experiment, the error is higher for the small

Table 1: Direction-only queries in simulator: evaluation of responses

Spatial Query	Precision	Recall	Accuracy
$fr(x, R)$	100.00%	84.62%	96.67%
$r(x, R)$	88.89%	94.12%	95.45%
$r\text{-}fr(x, R)$	90.00%	69.23%	92.42%
$r\text{-}b(x, R)$	66.67%	100.00%	98.48%
$l(x, R)$	94.12%	94.12%	96.97%
$l\text{-}fr(x, R)$	66.67 %	100.00%	95.45%
$l\text{-}b(x, R)$	81.82%	100.00%	96.97%

Table 2: Distance-only queries in simulator: evaluation of responses

| Spatial Query	Precision	Recall	Accuracy
$at(x, R)$	90.91%	90.91%	96.97%
$vc(x, R)$	100.00%	92.86%	98.48%
$c(x, R)$	80.00%	88.89%	95.45%
$f(x, R)$	84.62%	64.71%	87.88%
$vf(x, R)$	80.00%	92.31%	94.03%

Table 3: Queries combining distance and direction in simulator: evaluation of responses

Spatial Query	Precision	Recall	Accuracy
$fr(x, R) \wedge c(x, R)$	100.00%	83.33%	98.48%
$l(x, R) \wedge vc(x, R)$	85.71%	75.00%	95.45%
$r\text{-}fr(x, R) \wedge at(x, R)$	100.00%	71.43%	96.97%
$r(x, R) \wedge f(x, R)$	100.00%	100.00%	100.00%

regions of the qualitative direction ($l\text{-}fr$ and $l\text{-}b$).

The simulated robots work with the Cross Architecture concept (Perico et al. 2014), as well as the real robots. So, it is possible to program both simulated and real robots, in several languages, such as C++ and Python. This feature allowed us to extend the simulated research to our real humanoid robots without many changes. The simulated experiments were performed in an Intel i5 with 8GB SDRAM running Ubuntu 14.04[1].

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[1]The simulator used in this work, along with the source code of the proposal, are available at the URL http://fei.edu.br/~rbianchi/software.html
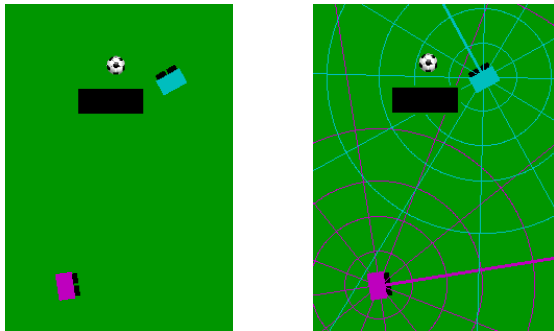
(a) Without visible discretisation   (b) With visible discretisation

Figure 8: RoboFEI-HT Soccer Simulator: one of the ball's position during the second experiment. Ball is always occluded from the red robot (R).

Table 4: Confusion matrix for regions of direction: ball's positioning inference w.r.t. to the red robot (R).

regions	r-fr	fr	l-fr	l	l-b	b
fr	7.8%	76.4%	15.8%	0.0%	0.0%	0.0%
l-fr	0.0%	10.0%	72.5%	17.5%	0.0%	0.0%
l	0.0%	0.0%	18.4%	73.7%	7.9%	0.0%
l-b	0.0%	0.0%	0.0%	20.4%	61.2%	18.4%

Table 5: Confusion matrix for regions of distance: ball's positioning inference w.r.t. to the red robot (R).

regions	at	vc	c	f	vf
at	86.6%	13.3%	0.0%	0.0%	0.0%
vc	3.3%	90.0%	6.6%	0.0%	0.0%
c	0.0%	0.0%	93.3%	6.6%	0.0%
f	0.0%	0.0%	6.6%	93.3%	0.0%
vf	0.0%	0.0%	0.0%	3.3%	96.7%

Real Robots

The experiments performed with real robots were conducted with humanoid robots inspired on the DARwIn-OP design (Ha et al. 2011). The robots have height of 490 mm; 20 degrees of freedom (6 for each leg, 3 for each arm and 2 in the neck) and a Full-HD camera located in the robot's head. On-board processing is made by an Intel NUC Core i5 with 8GB SDRAM. Figure 9a shows the robots in the first experiment.

Experiment 1: Communication Effectiveness with Three Robots. The effectiveness test with real robots considered the arrangement depicted in Figure 9, that is the reproduction of the arrangement shown in Figure 7. Such as executed in the simulator, the queries were made in relation to the red robot (R). The inferences were made by the blue robot (B) and by the yellow robot (Y). This test followed almost the same procedure adopted in the simulation, the difference is the number of times that each spatial query was repeated, that was 10 for the real robot.

Table 6 shows precision, recall and accuracy of the given answers, as well as Table 7 and 8. The vision system in real robots are lagged and noisy, so the errors can be even worse
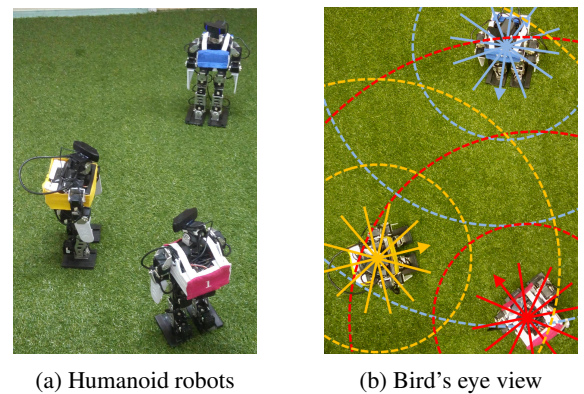
(a) Humanoid robots   (b) Bird's eye view

Figure 9: Robot's position for the first experiment.

Table 6: Direction-only queries in real robots: evaluation of responses

Spatial Query	Precision	Recall	Accuracy
$fr(x,R)$	76.00%	90.48%	80.00%
$r(x,R)$	–	–	100.00%
$r\text{-}fr$	–	–	87.50%
$r\text{-}b$	–	–	100.00%
$l(x,R)$	–	–	82.50%
$l\text{-}fr$	100.00%	75.00%	87.50%
$l\text{-}b$	–	–	100.00%

Table 7: Distance-only queries in real robots: evaluation of responses

| Spatial Query	Precision	Recall	Accuracy
$at(x,R)$	–	–	97.50%
$vc(x,R)$	94.74%	100.00%	97.37%
$c(x,R)$	80.00%	70.59%	80.00%
$f(x,r)$	–	–	100.00%
$vf(x,R)$	–	–	100.00%

Table 8: Queries combining distance and direction in real robots: evaluation of responses

Spatial Query	Precision	Recall	Accuracy
$fr(x,R) \wedge c(x,R)$	87.50%	77.78%	85.00%
$l(x,R) \wedge vc(x,R)$	–	–	100.00%
$r\text{-}fr(x,R) \wedge at(x,R)$	–	–	100.00%
$r(x,R) \wedge f(x,R)$	–	–	100.00%

than those one considered in the simulation. Even taking these errors into account, the lower precision found is 75% while the lower recall was around 70%. It is also possible to see in Tables 6, 7 and 8 that some precisions and recalls were not calculated, due to the fact that there were not true positives in the involved regions.

Experiment 2: Communication of Spatial Perceptions to Handle Occlusion. Since we have already had, from the simulation, a quantitative evaluation for the inference of an occluded ball, the second experiment was made in order to validate the results obtained in the simulation in the real robots. Moreover, this experiment aims to provide a comparison between the purely qualitative inference system of $\mathcal{OPRA}_m$ and this same qualitative inference system restricted by triangulation. Each part of the experiment was conducted with an orange ball in three distinct positions, while the blue robot (B) and the red robot (R) remained in the same position during all the experiment. The position of the spatial entities is depicted in Figure 10.

The first column of Table 9 represent the ground truth for this experiment. The results obtained by the qualitative-only inference and the results obtained by the qualitative location restricted by triangulation are shown in the second and third columns respectively. The grey regions represent the position of the ball – real, in the first column, and inferred, in second and third columns. It is worth noting that, in some situations, the purely qualitative inference executed by $\mathcal{OPRA}_m$ is not possible, such as the situation where the ball is positioned in 3 (Figure 10b), represented in the bottom line of Table 9. On the other hand, due to the inclusion of triangulation as a constraint for $\mathcal{OPRA}_m$ inference, the system
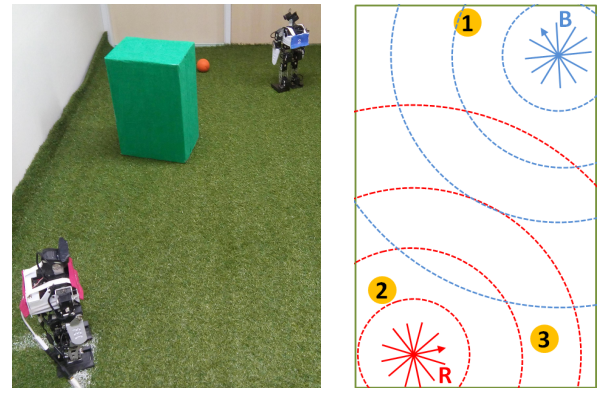
(a) Hidden ball.   (b) The 3 ball's positions.

Figure 10: The arrangement of the second experiment: the ball had 3 distinct positions while the robots keep their initial position.

was capable to infer the correct position.

Related Work

$\mathcal{OPRA}_m$ has been used as a method for integrating local knowledge in a quadruped mobile robot (Moratz and Ragni 2008), where, during the experiments, the robot was able to distinguish between colours and simple objects using a monocular vision system. Despite using computer vision, the robot had no prior knowledge of the size of objects. Thus, distance estimation was not possible; the only information available was the local orientation of the robot in relation to the objects. The robot was able to complete the task "move to the yellow cube behind the red disk" using the $\mathcal{OPRA}_6$ as an engine of reasoning. The task was transmitted to the robot by human speech commands. In another work, $\mathcal{OPRA}_4$ was applied to formalise the Inter-

Table 9: Inference of the ball's position wrt the red robot.

Ball Pos.	Ground Truth	Blue Inference	
		Purely Qualitative	Qualitative Restricted by Triangulation
1			
2			
3			

national Regulations for Preventing Collisions at Sea (Col-Regs) (Dylla et al. 2007). The authors focused their work on translating the navigation rules from natural language descriptions to a qualitative formalisation for agent control. $\mathcal{OPRA}$ calculus was chosen because direction information is extremely important in sea navigation. International Regulations for Preventing Collisions at Sea define, for each pair of vessels, which one has to give way and which is the privileged one, where different types of vessels require different rules. The qualitative representation was then obtained considering three vessels actions and the reasoning was made by constraint networks formed by the transition systems of the applicable rules. $\mathcal{OPRA}$ has also been used for navigation in street networks that are described via local observations (Lucke, Mossakowski, and Moratz 2011) and, when combined with the Region Connection Calculus (RCC-8) (Cohn et al. 1997), it has been used for defining the set of qualitative relations that represent social conventions (Dylla, Kreutzmann, and Wolter 2014).

The present work extends these qualitative methodologies applying them in real autonomous agents, where the distance concept was included. Also, by considering quantitative triangulation restricting $\mathcal{OPRA}_m$, the implementation described here shows better precision than its predecessors.

Conclusion

This work proposed a collaborative communication of spatial perceptions for multi-robot systems defined over $\mathcal{EOPRA}_m$ representation, where the reasoning is made by $\mathcal{OPRA}_m$ compositions combined with quantitative triangulations. This implementation was tested on a simulated environment and also on groups of real humanoid robots involved in collaborative tasks. Two experiments were considered: in the first, the group of robots had to answer spatial queries using the information perceived by each robot. This information was shared among the group members and the proposed inference was used to combine the multiple pieces of data, from where an answer to the query could be obtained by a simple predicate-unification process. In the second experiment, the proposed inference was used to communicate the observations of one robot about a target that was occluded with respect to another robot. The results obtained indicate that $\mathcal{EOPRA}_m$ representation is a suitable tool for representing (and sharing) qualitative spatial knowledge in groups of robots. Its qualitative nature allows for the definition of a small number of relations, that are closer to spatial predicates used in natural languages.

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